UniCredit Foundation

"Market Power and Innovation in the Intangible Economy"

Maarten De Ridder

2020 CopenhagenMacroDays Best Paper Award

Working Paper Series

n. 142 ■ September 2020

Statement of Purpose

The Working Paper series of the UniCredit Foundation is designed to disseminate and to provide a platform for discussion of either work of the UniCredit economists and researchers or outside contributors (such as the UniCredit Foundation scholars and fellows) on topics which are of special interest to the UniCredit Group. To ensure the high quality of their content, the contributions are subjected to an international refereeing process conducted by the Scientific Committee members of the Foundation.

The opinions are strictly those of the authors and do in no way commit the Foundation and UniCredit.

These Working Papers often represent preliminary work. Citation and use of such a paper should take account of its provisional character.

The Working Papers are also available on our website (http://www.unicreditfoundation.org)

Market Power and Innovation in the Intangible Economy*

Maarten De Ridder‡
London School of Economics

First version: 20 March 2019. This version: 8 September 2020

Abstract

This paper offers a unified explanation for the slowdown of productivity growth, the decline in business dynamism and the rise of market power. In a quantitative framework, I show that the rise of intangible inputs – such as software – can explain these trends. Intangibles reduce marginal costs and raise fixed costs, which gives firms with high-intangible adoption a competitive advantage, in turn deterring other firms from entering. I structurally estimate the model on French and U.S. micro data. After initially boosting productivity, the rise of intangibles causes a significant decline in productivity growth, consistent with the empirical trends observed since the mid-1990s.

Keywords: Productivity, Growth, Business Dynamism, Intangible Inputs, Market Power

*I am grateful to Vasco Carvalho for extensive feedback and valuable discussions. I am also grateful for frequent feedback from Giancarlo Corsetti, Bill Janeway, Damjan Pfajfar and Coen Teulings, as well as for discussions with Klaus Adam, Andy Atkeson, Ariel Burstein, David Baqaee, Tiago Cavalcanti, Chiara Criscuolo, Luca Dedola, Wouter den Haan, Romain Duval, Oliver Exton, Emmanuel Farhi, Lukas Freund, Jonathan Haskel, Marie Hoerova, Hugo Hopenhayn, Peter Karadi, Joseba Martinez, Sophie Piton, Pontus Rendahl, Alex Rodnyansky, Christina Romer, David Romer and Petr Sedláček. I thank Jean Imbs and especially Basile Grassi for their help with data access and Isabelle Mejean for help with merging French datasets. I thank participants at the Bank of England, Berkeley Haas, Bocconi, Brown, Cambridge, CPB, CREi, Dallas Fed, ECB, EIEF, IIES, Kent, LSE, Maryland, SSE, UCLA, UCSD, UPF, USC, Zurich, and the 7th Joint CBRT-ECB Conference for comments and discussion. This work is supported by a public grant overseen by the French National Research Agency (ANR) as part of the 'Investissements d'Avenir' program (reference: ANR-10-EQPX-1, CASD). Financial support from the Prins Bernhard Cultuurfonds and the Keynes Fund is gratefully acknowledged.

‡*Address:* London School of Economics, Department of Economics, 32 Lincoln's Inn Fields, U.K. *E-mail:* maartenderidder92@gmail.com. *Web:* http://www.maartenderidder.com/.

1. Introduction

The decline of productivity growth has played a prominent role in recent academic and policy debates. Average productivity growth in the United States was less than 0.5% between 2005 and 2018, well below the long-term average of 1.3% (Figure 1a). A similar slowdown occurred across most of Europe, causing productivity in countries such as France and the United Kingdom to flatline (Adler et al. 2017). The slowdown followed after a decade of above-average growth, fueled by rapid improvements in information technologies (Fernald 2015). The slowdown occurred despite an increase in productivity-enhancing investments: U.S. investments in corporate research and development have increased by 65% as a fraction of national income over the last 30 years (Figure 1b). The slowdown therefore does not seem to be driven by a lack of effort to become more productive, but rather by a decline in the effect of innovative investments on productivity growth.[1]

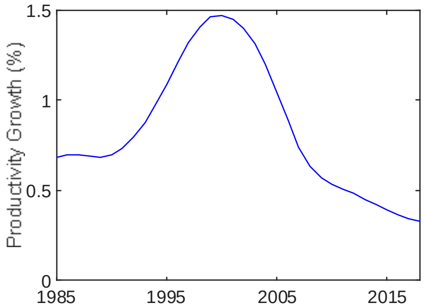
The initial surge and subsequent decline in productivity growth coincided with two other trends: the slowdown of business dynamism and the rise of markups. Signs that dynamism is weakening include the decline in the rate at which workers reallocate to different firms (e.g. Decker et al. 2014), the decline in skewness of the firm-growth distribution (e.g. Decker et al. 2016) and the decline of entry rates (e.g. Pugsley and Şahin 2018). The rise of markups has recently attracted attention and has been linked to the decline of the labor share (e.g. De Loecker et al. 2020). Despite the growing body of evidence detailing these trends, there is thus far no consensus on what has caused them.

This paper claims that the trends in productivity growth, business dynamism and markups can jointly be explained by a secular shift in the way firms produce. Specifically, I show that an increase in the use of intangible inputs can drive these patterns. Intangible inputs are inputs that are used in production, but that are not physically embodied. Information technology and software are prominent examples. The rise of intangible inputs has been dramatic over the last 30 years: software alone is now responsible for 18% of U.S. corporate investments, up from 3% in 1980 (BEA).

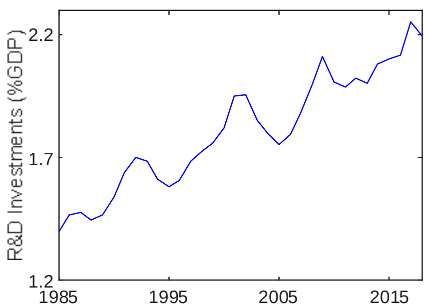
Intangible inputs can explain the three trends because they have two features: they are scalable, and firms differ in the efficiency with which they deploy them. Intangibles are scalable in the sense that they can be duplicated at close-to-zero marginal cost (e.g. Haskel and Westlake 2017, Hsieh and Rossi-Hansberg 2019). This causes the cost structure to change when firms use intangible inputs in production. Firms invest in the development and maintenance of intangible inputs but face minimal additional costs when production is scaled up. An example of such an input is Enterprise Resource Planning (ERP), which firms use to automate business processes such as supply chain and inventory management. ERP allows firms to automatically send invoices or order supplies, for example, which reduces the marginal cost of a sale. Alternatively, firms that sell products that include software (e.g. the operating system of a phone, a car's drive-by-wire-system) face minimal costs of reproducing software in additional units. The rise of intangibles therefore shifts costs away from the marginal towards the fixed component.

[1] Bloom et al. (2020) show that the aggregate effect of innovative efforts on growth is falling. They document declines in the effectiveness of research in firm-level data and various case studies, such as the effort needed to double the power of computer chips (Moore's Law), agricultural productivity and pharmaceutical innovation.

Figure 1. Trends in Productivity Growth and Research & Development



(a) Productivity Growth



(b) R&D Investments

Notes: Figure 1a plots annual productivity growth from the Fernald series (FRBSF). The plot is smoothed using an HP filter with an annual smoothing parameter of 100. Figure 1b plots R&D as a percentage of GDP. Data is from the BEA NIPA tables.

Firms differ in the extent to which they adopt intangible inputs to reduce their marginal costs. A 2018 European Investment Bank survey finds that over 40% of American and European manufacturing firms do not use state-of-the-art digital technologies, while less than 15% organize their entire operation around digital technologies (Veugelers et al. 2020).[2] A likely driver is the fact that firms, even within narrowly defined industries, differ in the efficiency with which they can reduce their marginal costs through intangibles. A rich literature provides evidence on this. Bloom et al. (2012), for example, show that American-owned European establishments achieve greater productivity improvements from the use of information technology (IT). They find that intangible input productivity is a firm characteristic, especially because the IT productivity of European establishments increases when they are *acquired* by an American firm. Schivardi and Schmitz (2019) furthermore show that inefficient management practices can explain not only the low IT adoption by Italian firms but also why the productivity gains that these firms obtain from using IT are limited.[3]

I show that intangible inputs modeled along these lines can qualitatively and quantitatively explain the trends in productivity growth, business dynamism and markups.[4] To do so, I introduce intangible inputs in an endogenous growth model in the spirit of Klette and Kortum (2004) that is tractable yet sufficiently rich to quantitatively analyse the effect of intangibles. Each firm produces one or multiple goods and invests in research and development (R&D) to create higher-quality versions of goods that other firms produce. Successful innovation causes the innovator to become the new producer, while the incumbent ceases to produce the good. Step-wise improvements to random goods through this process of creative destruction are the driver of aggregate growth.

[2] A full literature review on firm-level determinants of IT adoption is provided in Haller and Siedschlag (2011).

[3] Bloom et al. (2014) also find that structured management practices are closely related to IT adoption in American firms. Evidence also suggests that workplace organization and organization capital affect a firm's IT productivity (e.g. Crespi et al. 2007, Bartel et al. 2007). Changes to organization design come at the price of high adjustment costs, which makes IT productivity a persistent firm characteristic (e.g. Bresnahan et al. 2002).

[4] Software and information technology are used throughout this paper as examples of inputs that are scalable and deployed at heterogeneous efficiency, as investments in these inputs have increased rapidly over the last 30 years. Any other input, however, that satisfies both requirements could be used to explain the trends in productivity growth, business dynamism and markups in the framework.

Intangible inputs enter the model through the production function. Firms are able to reduce their marginal costs by committing to the purchase of fixed-cost intangibles. As firms differ in the efficiency with which they deploy intangibles, some firms choose to reduce their marginal costs by a greater fraction than others. This introduces a new trade-off between *quality* and *price* to the Klette and Kortum (2004)-framework. In the standard model, firms that develop a higher quality version of a good become its sole producer. Other firms have the same marginal costs but are unable to produce the same quality and hence cannot compete. Intangible inputs change this result, as high-intangible firms are able to produce their output at lower costs than others. Their cost-advantage allows them to sell at lower prices. When a firm with a lower level of intangible-adoption develops a higher quality version of a good sold by one of these firms, the incumbent could undercut the innovator on price. Only if the quality difference is sufficiently large to offset the gap in marginal costs would the innovator become the new producer. The presence of firms with a high take-up of intangible inputs, therefore, deters other firms from entering new markets. The rise of firms with high-intangible productivity can therefore negatively affect productivity growth.

To analyze whether this explains the macroeconomic trends, I introduce high-intangible entrants to an economy where firms initially use similar levels of intangibles. Over the transition path, the rise of high-intangible firms initially causes a boom in productivity growth. As they have a greater incentive to invest in R&D, they serve to "disrupt" sectors, and economic activity concentrates disproportionately around these firms. Their entry raises productivity because high-intangible firms produce all their goods at a lower cost. The increase in aggregate productivity is not matched by wages, however, because high-intangible firms set proportionally higher markups. As the economy transitions to the new balanced growth path, there is a decline in entry as most start-ups are unable to compete with high-intangible incumbents. Low-intangible incumbents similarly have weaker incentives to innovate. This causes a gradual decline in productivity growth, which falls below the initial steady-state level around 20 years after the first high-intangible firms enter the market. Although overall R&D increases, it concentrates around a smaller group of firms. Because returns are concave, the concentration of R&D lowers its effectiveness. Combined with the fact that a fraction of innovations fail because high-intangible incumbents undercut innovators on price, this explains how growth can fall while innovative investments increase.

I quantify the model using two structural estimations, one for the U.S. and one for France. The French estimation relies on the administrative data for the universe of firms while that for the U.S. relies on data for listed firms. While evidence on the macroeconomic trends is stronger for the U.S., I show that the trends are largely visible for France as well. The advantage of French data is that the full income statement and balance sheet are available for both public and private firms and that it can be merged with surveys on innovation activities and the adoption of IT systems. This allows a close inspection of the empirical validity of the model's mechanisms. Using a new measure of fixed costs, I show that the share of fixed costs in total costs gradually increased from 14 to 24.5% in the U.S. between 1980 and 2016 and from 9.5 to 14% in France between 1994 and 2016. There is a positive within- and across-firm correlation between fixed costs and investments in software, as well as the adoption of intangible inputs such as ERP. Firms with a high fixed-cost share also invest more in R&D and have higher average growth rates, in line with the model's predictions.

The structurally estimated model explains a significant part of the slowdown of productivity growth, the decline in business dynamism and the rise of markups. The model predicts a slowdown in steady-state growth of 0.43 percentage points in the U.S. calibration and of 0.23 percentage points in the French calibration, after an initial boom in growth of six years. Markups increase by 21.8 and 22.9 percentage points in the respective calibrations. The entry rate falls by 5.8 and 4.5 percentage points, respectively. If markups are assumed to be constant, the model predicts a greater decline in productivity growth and business dynamism. The rise of markups stimulates innovative investments by high-intangible firms, and therefore mitigates the decline in productivity growth and business dynamism in the baseline model.[5]

Besides explaining the macroeconomic trends, the model offers two theoretical insights. First, it shows that the effect of R&D on aggregate growth depends on on how R&D is distributed across firms. Because firm-level returns are concave in expenditure, concentration of R&D negatively affects growth. Firm heterogeneity is therefore an important ingredient for this type of model. Second,it introduces a distinction between quality and price. Productive (high-intangible) firms are able to sell at lower prices, which can compensate for lower quality and can be used to undercut innovators. Differences in efficiency across firms therefore reduce the effect of R&D on growth.

Related literature This paper relates most closely to recent work that jointly explains low productivity growth, the fall in business dynamism and the rise of market power. In Aghion et al. (2019), the rise of IT increases the span of control, which allows ex-ante productive firms to grow larger. Because productive firms become more likely to face productive competitors, expected markups fall and incentives to innovate decline. This reduces R&D and subsequent growth. My model predicts an *increase* in aggregate R&D, but because it is concentrated among a smaller group of highly profitable, high-intangible firms, there is a decline in aggregate growth. Peters and Walsh (2019) relate the decline of entry to the fall in labor force growth. The lack of entry stimulates expansion by incumbents, which raises firm concentration and markups and slows down productivity growth. Liu et al. (2019) relate productivity and business dynamism to low interest rates. Low interest rates increase investment most strongly for the market leader, discouraging investments by the follower and diminishing growth. Akicigit and Ates (2019) find that intellectual property rights are increasingly used anti-competitively, which also discourages entry. Both forms of discouragement differ from mine, as my discouraging effect arises from the inability of low-intangible firms to compete on price. My framework also predicts that the rise of intangibles initially causes a boom in growth.[6]

[5] This paper's analysis is therefore robust to concerns about the firm-level measurement of markups (Traina 2018, Bond et al. 2020) and recent evidence that the labor share is stable outside of the U.S. (Gutierrez and Piton 2020).

[6] Cavenaile et al. (2020) extend the Schumpeterian growth model with detailed oligopolistic competition. They structurally estimate their framework on data before and after the slowdown of productivity growth, and find that the cost parameters for innovation are higher in the latter period. My model provides an explanation for the rising cost of innovation, as low-intangible firms are less likely to successfully innovate for a given spending on R&D.

Other papers explain a subset of the macroeconomic trends. Hsieh and Rossi-Hansberg (2019) suggest that intangibles explain the rise of concentration in services, as software can be deployed across markets after paying a fixed cost. My paper is complementary, as it shows that fixed-cost intangibles can also explain the slowdown of productivity growth. Brynjolfsson et al. (2020) claim that adopting artificial intelligence requires unmeasured investments, causing measured productivity to initially decline but eventually to increase. Hopenhayn et al. (2018) argue that the decline of labor force growth explains most of the fall in business dynamism. Weiss (2019) relates intangibles to the rise of industry concentration and markups. Korinek and Ng (2019) and Martinez (2019) respectively relate automation to the rise of concentration and to the decline of the labor share.

The theoretical framework builds on Schumpeterian growth models of creative destruction in the tradition of Aghion and Howitt (1992). It is part of the strand of Schumpeterian models where firms produce multiple products (Klette and Kortum 2004). This framework is attractive because it is analytically tractable, yet able to replicate many empirical features of firm dynamics (Lentz and Mortensen 2008). The framework was recently used to study the reallocation of innovative activity (Acemoglu et al. 2018), to discern the effect of innovation policy (Atkeson and Burstein 2019) and to compare different sources of innovation (Akcigit and Kerr 2018, Garcia-Macia et al. 2019). It has also been used to analyze misallocation in a setting with heterogeneous markups (Peters 2019).

This paper also relates to the recent literature that studies the trends in productivity and market power from a disaggregated perspective. As summarized by Van Reenen (2018), there is substantial heterogeneity in the extent to which firms are subject to these trends, causing productivity and profitability to diverge. Andrews et al. (2016) show that productivity growth of the most productive firms has not declined. Decker et al. (2018) find an increase in productivity dispersion within the U.S.[7] The rise in markups in De Loecker et al. (2020) is also strongest in the highest deciles, a result that has been confirmed for several countries (Diez et al. 2019, Calligaris et al. 2018). Recent summaries of the debate on markup estimation methodologies are found in Syverson (2019), Basu (2019), and Bond et al. (2020). I show that an increase in the ability to use intangibles by some firms can impose a negative externality on others, thereby driving the growing differences across firms as well as the aggregate trends in productivity growth, business dynamism and markups.

More broadly, this paper relates to work on the rise of corporate profits. Barkai (2020) finds that excess profits have increased over time because payments to labor and capital have declined as a percentage of GDP. Caballero et al. (2017) remark that this is partly offset by a rise in risk premia. Karabarbounis and Neiman (2019) add that unmeasured capital also explains the rise of excess profits, which they refer to as factorless income. Gutiérrez and Philippon (2019) show that the response of entry to profitability of incumbents has declined over time. Gutierrez and Philippon (2017) further relate the lack of investments relative to Tobin's Q to a decline in competition.

A related literature measures the static costs of markups. Edmond et al. (2019) find that markups reduce welfare by 7.5%. Baqaee and Farhi (2020) argue that markups reduce TFP by 20% and find that the rise of aggregate markups is driven by reallocation of economic activity towards high-

[7] Kehrig and Vincent (2019) note that an increase in productivity dispersion at the establishment level may reflect an improvement in factor allocation and a reduction of internal credit market frictions.

markup firms. Their result is in line with the finding in Autor et al. (2020) and Kehrig and Vincent (2020) that the decline in the labor share is driven by a reallocation of activity towards firms with a low labor share. My model similarly predicts a reallocation towards high-markup (high-intangible) firms, as these have a greater incentive to expand by investing in R&D.

My theoretical predictions are in line with empirical work that relates productivity growth, business dynamism and market power to intangibles. Crouzet and Eberly (2018) show that intangibles cause an increase in market power and productivity for leading U.S. public firms. McKinsey (2018) and Ayyagari et al. (2018) show that firms with high profitability and growth invest more in software and R&D. Bessen and Righi (2019) find that productivity of U.S. firms increases persistently after an increase in the stock of their IT staff. Farhi and Gourio (2018) show that unmeasured intangibles can explain the rising wedge between the measured marginal product of capital and risk-free rates. Bajgar et al. (2019) find that sectors with high intangible investments experienced a greater increase in concentration. Bessen (2017) finds a positive sector-level relationship between concentration and the use of IT systems, and stresses that the scalability of intangibles is advantageous to firms that are already large. Firm-level evidence on this is provided in Lashkari et al. (2019). Calligaris et al. (2018) find a positive correlation between the use of digital technologies and the rise of markups. Bijnens and Konings (2018), documenting a decline in Belgian business dynamism, remark that the decline is strongest in industries with a high IT intensity.

Outline The remainder of this paper proceeds as follows. Section 2 introduces fixed-cost intangible inputs empirically. Section 3 presents the growth model and discusses the main mechanism. The model is structurally estimated in Section 4, and results are discussed in Section 5. Section 6 presents extensions, while Section 7 concludes.

2. Intangibles as Fixed Costs

This section introduces intangibles as inputs that cause a shift from marginal to fixed costs. To provide a foundation for the main analysis, I outline a simple framework where intangibles are modeled as such an input. I then present micro evidence on two facts that are consistent with this framework: the share of fixed costs increases over time, and there is a positive correlation between fixed costs and either software investments or measures of information technology adoption.

2.1. Framework

Consider a first-degree homogeneous production function $z(z_{it,1}, z_{it,2}, .., z_{it,k}) \cdot \omega_i$ with k traditional (tangible) production factors and Hicks-neutral productivity ω_{it}. Firm i's marginal cost function is $\boldsymbol{c}(w_{1t}, w_{2t}, .., w_{kt}, \omega_{it})$, where w_{kt} denotes the factor price of tangible production factor k at time t. Intangible inputs are defined as inputs that allow firms to reduce their marginal costs by a desired fraction $s_i \in [0,1)$.[8] In the framework, the production function therefore reads

$$y_{it} = \frac{1}{1-s_{it}} \cdot z(z_{it,1}, z_{i,2}, .., z_{it,k}) \cdot \omega_{it}, \tag{1}$$

which is associated with marginal costs $mc_{it} = (1-s_{it}) \cdot \boldsymbol{c}(w_{1t}, w_{2t}, .., w_{kt}, \omega_{it})$.[9] To reduce their marginal costs by s_{it}, firms must spend some amount on intangible inputs. The relationship between s_{it} and expenditure on intangibles is governed by a twice-differentiable function $f(s_{it}, \phi_i)$. ϕ_i is a firm-specific parameter that captures the efficiency with which firm i uses intangibles: firms with higher levels of ϕ_i are able to reduce their marginal costs by a greater fraction for a given expense on intangible inputs. $f(\phi_i, s_{it})$ is strictly convex on the domain $s_{it} \in [0,1)$ and satisfies $\partial f(\phi_i, s_{it})/\partial\phi_i < 0$, $f(\phi_i, 0) = 0$ and $\lim_{s_{it}\to 1} f(\phi_i, s_{it}) = \infty$. The latter implies that the cost of eliminating marginal costs completely is infinite, such that all firms have positive marginal costs in equilibrium. Firms pay $f(\phi_i, s_{it})$ before production occurs; this, combined with the fact that $f(\phi_i, s_{it})$ does not directly depend on the amount that a firm sells, explains why they represent a fixed cost.[10] The term "fixed" here is different from usual, in the sense that firms choose the level of $f(\phi_i, s_{it})$ through intangible inputs. Firms that do not increase their use of intangibles do not face an increase in fixed costs, and intangibles do not directly raise entry costs. Total costs tc_{it} equal

$$tc_{it} = (1-s_{it}) \cdot \boldsymbol{c}(w_{1t}, w_{2t}, .., w_{kt}, \omega_{it}) \cdot y_{it} + f(s_{it}, \phi_i),$$

where the first term contains all variable costs while the second term contains fixed costs. It is straightforward to show that when firms increase their expenditure on intangibles there is a shift from variable to fixed costs, provided that a reduction in marginal costs does not lead to a large increase in demand. Formally, $\partial f(s_{it})/tc_{it}/\partial s_{it} > 0$, provided that

$$\frac{\partial \ln z(z_{it,1}, z_{it,2}, .., z_{it,k})}{\partial s_{it}} < 1. \tag{2}$$

Under this condition, which I view as mild, the rise of intangible inputs is reflected by an increase in the average share of fixed costs in total costs and this share should increase at the firm level when firms increase their use of intangible inputs.

2.2. Data

To test the empirical validity of the framework, I use data from financial statements on U.S. publicly listed firms and administrative data on the universe of French firms. Appendix D, replicating the

[8] This definition applies to a subset of the total of possible intangible assets and inputs that firms may deploy. It might not apply, for example, to research and development expenses, which are treated separately in the model in Section 3. Throughout the text, the term 'intangible inputs' refers to inputs for which the definition applies.

[9] Instead of dividing by $1-s_{it}$ one could multiply $z(\cdot)$ by a productivity term that depends on intangibles. That approach is isomorphic to my approach, which I prefer because it leads to a convenient expression for marginal costs.

[10] This does not mean that there is no correlation between $f(\phi_i, s_{it})$ and output, as large firms have greater incentives to reduce marginal costs and choose a higher $f(\phi_i, s_{it})$. The empirical analysis therefore includes controls for size.

Table 1: Descriptive Statistics

	Mean	Std. Dev.	Median	10th Pct.	90th Pct.	Obs.
U.S. Compustat Firms (1980-2016)						
Sales (revenue)	2,370,409	14,147,340	189,447	13,237	3,521,189	125,231
Operating expenses	2,011,057	12,196,450	164,736	12,767	2,990,300	125,231
Cost of goods sold	1,596,605	10,677,900	113,106	7,007	2,290,489	125,231
Selling, General, and Adm. expenses	414,452	1,970,337	39,066	3,857	635,171	125,231
Capital stock	1,567,708	10,485,090	77,258	4,889	2,072,819	125,231
French Firms in FICUS-FARE (1994-2016)						
Sales (revenue)	4,684	103,285	617	149	4,996	9,913,058
Employment (headcount)	19	356	5	1	28	9,913,058
Wage bill	622	10,753	144	38	831	9,913,058
Capital stock	1,738	131,183	92	12	895	9,913,058
Intermediate inputs and raw materials	2,234	58,699	136	0	1,923	9,913,058
Other operating expenses	1,210	35,652	124	33	1168	9,913,058

Notes: Nominal figures in thousands of Dollars (U.S.) and Euros (France). Sales, operating costs and materials are deflated with KLEMS sector deflators; the wage bill and capital are deflated with the GDP deflator.

macroeconomic trends that motivate this paper for France, confirms that it has incurred a decline in productivity growth and business dynamism, as well as a modest increase in markups.

Data for U.S. firms is obtained from S&P's Compustat. Compustat contains balance sheet and income statement data for all publicly listed firms in the U.S. I restrict the sample to firms outside of finance, insurance and real estate between 1980 and 2016, and drop firms with missing or negative sales, assets and operating expenses. Following Baqaee and Farhi (2020), I drop firms with ratios of sales to cost of goods sold or of sales to selling, general, and administrative expenses outside of the 2.5-97.5 percentile range. The sample covers 10,738 firms across 788 6-digit NAICS industries.

The French data come from two administrative datasets (FICUS, from 1994 to 2007, and FARE, from 2008 to 2016), both based on tax data from DGFiP. The data contain the full balance sheet and income statement, with detailed breakdowns of revenues and costs. I append FICUS with FARE using a firm identifier (the *siren code*) that consistently tracks firms over time. The unit of observation is a legal entity (*unité légale*), although subsidiaries of the largest companies are grouped as a single entity. I restrict the sample to private firms, and drop contractors, state-owned enterprises and non-profit organisations, as well as companies that receive operating subsidies in excess of 5% of sales. Firms in financial industries and firms with missing or negative sales, assets or employment are also excluded. Details on variable definitions are provided in Appendix B. The remaining sample contains data on 1,087,726 firms across 651 NACE industries between 1994 and 2016.[11]

Summary statistics for both datasets are provided in Table 1.

2.3. Measurement and Analysis

Testing the framework requires a measure of fixed costs. Past work typically infers fixed costs from the sensitivity of a firm's operating costs or profits to sales shocks, under the assumption that all

[11]Access to the FICUS and FARE datasets was initially obtained for Burstein et al. (2019). The code to merge FICUS and FARE was developed for their project, and is partly provided by Isabelle Mejean. I thank them for their help in obtaining data access and for permission to use the data for this project.

variable costs are set freely.[12] This is problematic when firms face adjustment costs for some variable inputs (when adjusting the size of their labor force, for example). I therefore derive a new time-varying measure of fixed costs from the difference between the marginal cost markup and the profit rate, which equals operating profits over revenue. Under the first-degree homogeneity assumption of $z(z_{it,1},..,z_{it,k})$, the accounting definition for the profit rate is

$$\frac{\pi_{it}}{p_{it}\cdot y_{it}} = \frac{\left(p_{it} - mc_{it}\right)\cdot y_{it}}{p_{it}\cdot y_{it}} - \frac{\tilde{f}_{it}}{p_{it}\cdot y_{it}},$$

where fixed costs $\tilde{f}_{it}$ are the sum of expenditures on intangibles and other fixed costs (η_i), such that $\tilde{f}_{it} = f(s_{it},\phi_i) + \eta_{it}$. Isolating fixed costs and defining the markup μ_{it} as the ratio of prices to marginal costs yields

$$\frac{\tilde{f}_{it}}{p_{it}\cdot y_{it}} = \left(1 - \frac{1}{\mu_{it}}\right) - \frac{\pi_{it}}{p_{it}\cdot y_{it}}. \tag{3}$$

I multiply the right-hand side of (3) with revenues and divide by total operating costs to obtain fixed costs as a share of total costs. The straightforward intuition behind (3) is that *markups* capture the firm's marginal profitability, while *profits* capture the firm's average profitability. Because fixed costs are incurred regardless of sales, a firm with positive fixed costs should have a profit rate below the markup. This implies that rising markups do not necessarily reflect rising profitability.

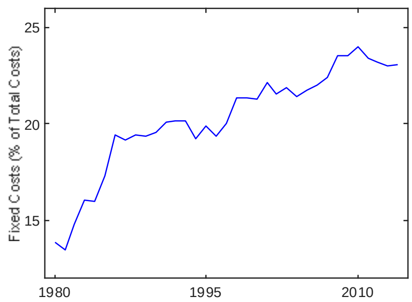
To implement the measure in equation (3), I require data on operating profits, revenues and markups. Operating profits and revenues are obtained from the income statement. Markups are not directly observed because income statement and balance sheet data lack information on marginal costs and prices. Instead, I estimate markups using the method proposed by Hall (1988). He shows that markups are given by the product of the output elasticity of a variable input multiplied by the ratio of a firm's sales to its expenditure on that input. Sales and expenditure on the input are observed on the income statement, while I obtain the output elasticity by estimating a translog production function using the procedure proposed by De Loecker and Warzynski (2012).[13]

Figure 2 depicts the sales-weighted average ratio of fixed to total costs as measured along equation (3). In line with trends in intangibles such as software, the measure shows a persistent increase in both France and the U.S. Fixed costs made up 13.9% (9.5%) of costs for American (French) firms at the start of the sample, and close to 24.5% (14%) at the end. Over the full episode there is a greater increase in fixed costs for U.S. firms, but this seems to be due to the difference in time samples. Between 1995 and 2015, firms in both datasets have an average increase in the fixed-costs share of

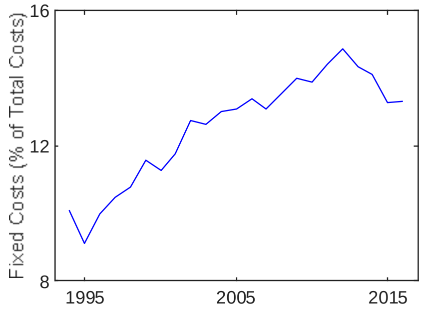
[12] Examples include Lev (1974) and García-Feijóo and Jorgensen (2010). Alternatively, De Loecker et al. (2020) assume that selling, general and administrative expenses on the income statement are fixed. Though appropriate for their purpose, it is likely that some of these costs (such as shipping costs and sales commissions) are variable.

[13] Details are provided in Appendix C. The advantage of this approach to estimating markups is that it does not assume any form of market structure or competition, and is consistent with the framework in Section 2.1. Furthermore, markups are estimated based on a single variable input m. Other inputs may be fixed, variable or a combination of both: as long as one freely-set variable input is observed, markups can be estimated consistently.

Figure 2. Weighted-Average Ratio of Fixed Costs to Total Costs



(a) **United States**



(b) **France**

Notes: Sales-weighted average of fixed costs as a percentage of total costs, U.S. listed firms (left) and universe of French firms (right). Fixed costs are inferred from the difference between profits as a percentage of sales and the marginal cost markup.

approximately 5 percentage points.[14] Appendix C shows that the trend in fixed costs is robust to alternative estimates for the markup. The appendix also contains an illustration of the sectoral composition of fixed costs (Figure A2). It shows that fixed costs are especially high in the information sector, while variable costs are relatively important in retail and wholesale. Nearly all broad sectors have seen an increase in their share of fixed in total costs, and a formal between-within decomposition in Table A2 confirms the increase in fixed costs occurs largely within sectors.

I next assess the relationship between the rise of fixed costs and the rise of intangible inputs. The framework in Section 2.1 implies that firms with higher intangible inputs should have greater fixed costs as a fraction of total costs, and that this fraction should increase when firms make additional investments in software. This can be tested using the French data, as it contains various measures of investments in software and information technology. The additional data comes from two surveys that are based on a (post-weighted) representative sample. The first is the *Enquête Annuelle d'Entreprises* (EAE), which is an annual survey of around 12,000 firms between 1994 and 2007. The survey provides a comprehensive panel of firms with more than 20 employees, and samples smaller firms in most sectors. I use this survey to obtain the amount that firms spend on software, either developed in-house or purchased externally.[15] The estimation equation reads:

$$\frac{\tilde{f}_{it}}{tc_{it}} = \alpha_i + \psi_t + \gamma \cdot \frac{f_{it}}{p_{it} \cdot y_{it}} + \beta' g(p_{it} \cdot y_{it}) + \varepsilon_{it},$$

where f_{it} is observed software in Euro, g is a polynomial of size controls, while α_i and ψ_t respectively denote firm- and time-fixed effects. Fixed effects are feasible because the full coverage of larger firms gives a sufficiently large panel. Results are presented in Table 2. Observations are weighted by their sample weights and variables are winsorized at their 1% tails. The table

[14] The level of the fixed-cost measure mostly depends on the estimate of the supply elasticity that is used to calculate markups. Some estimations of these elasticities are consistently lower than the level used for fixed costs in Figure 2, and therefore imply a lower level of fixed costs. The trend was similar across estimations, however. Appendix C contains a full robustness check of all results in this section using different production function estimates.

[15] This survey was also used to measure software by Lashkari et al. (2019). Details are provided in Appendix B.

Table 2: Relationship between Software Spending and Fixed-Cost Share (France)

Fixed-Cost Share	I	II	III	IV	V	VI
Software Investments	5.60***	5.19***	3.03***	2.69***	1.45***	0.55***
	(0.235)	(0.235)	(0.242)	(0.242)	(0.138)	(0.127)
Year fixed effects	No	Yes	No	Yes	No	Yes
Firm fixed effects	No	No	No	No	Yes	Yes
Industry fixed effects	No	No	Yes	Yes	No	No
Size Poly.	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.125	0.132	0.289	0.295	0.073	0.196
Observations	136,208	136,208	136,208	136,208	136,208	136,208

Dependent variable is fixed costs as a percentage of total costs. Explanatory variable is software investments as a percentage of sales. Sales is deflated with the sector-specific gross output deflator, software with the investment input deflator from EU-KLEMS. Firm-clustered standard errors in parentheses. *, **, *** denote significance at the 10, 5 and 1% levels, respectively.

shows a consistently positive relationship between software investments and fixed costs, though the strength of the relationship depends on the inclusion of fixed effects. The latter may be due to the fact that only firms with more than 20 employees are sampled more than once. The fact that the positive relationship is also present when controlling for firm-fixed effects suggests that fixed costs increase when firms increase their use of software. This supports the assumption to model intangible inputs as endogenous fixed costs in production. The coefficients in Table 2 are economically significant: a firm that moves from the median to the 95th percentile of software investments increases its fixed-cost share by 0.4 (column VI) to 4 (column I) percentage points.

There is also a positive relationship between fixed costs and the adoption of specific information technologies. Data comes from the *Enquête sur les Technologies de l'Information de la Communication* (TIC), a survey on the use of IT systems from 2008 to 2016 which covers an annual sample of around 10,000 firms with at least ten employees. The estimation equation reads

$$\frac{\tilde{f}_{ijt}}{tc_{ijt}} = \alpha_j^h + \psi_t^h + \gamma^h \cdot T_{ijt}^h + (\beta^h)' g(p_{ijt} \cdot y_{ijt}) + \varepsilon_{ijt}^h,$$

where T_{ijt}^h is a dummy that equals one if firm i in 5-digit industry j has adopted technology h. The TIC samples different firms each year, except when firms have been sampled multiple times.

Table 3: Relationship between Technology Adoption and Fixed-Cost Share (France)

	Software Adoption					
Fixed-Cost Share	ERP	CRM	CAD	SCM	RFID	Spec.
Adoption Dummy	0.015***	0.006***	0.020***	0.004	0.023***	0.045***
	(0.002)	(0.002)	(0.006)	(0.003)	(0.006)	(0.004)
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Industry fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Size Poly.	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.320	0.319	0.317	0.346	0.385	0.355
Observations	63,928	69,200	30,415	45,685	16,847	46,806

Notes. Explanatory variable is a dummy for the adoption of the technology specified in the column header (details provided in main text). Industry-fixed effects at the 5-digit NACE level. Firm-clustered standard errors in parentheses. *, **, *** denote significance at the 10, 5 and 1% levels, respectively. Observation counts differ, as not every measure was included in each survey year.

This is mainly the case for large firms, which makes the sample unrepresentative as a panel. The specification therefore includes industry- rather than firm-effects. Though the TIC contains various measures of technology adoption, I focus on five technologies that are available for a number of years and that are likely to capture s_{it}. Table 3 presents the results. The top of each column presents the technology used for T^h_{ijt}. ERP refers to enterprise resource planning, CRM to customer resource management, CAD to computer-aided design, SCM to supply chain management software and RFID to radio frequency identification. The explanatory variable in the final column is a dummy that equals one if the firm employs IT specialists. Observations are weighted by their sample weights. Except for SCM, they have a strong correlation with the share of fixed costs. The estimates are economically significant: a firm that uses ERP, on average, has a fixed cost ratio that is 1.5 percentage points higher than similarly-sized firms in the same 5-digit industry not using ERP. These results confirm that there exists a positive relationship between the adoption of intangible technologies and a firm's ratio of fixed costs to total costs.

3. Intangibles, Firm Dynamics and Growth

The previous section introduced intangibles as an input that raises fixed costs. In this section I introduce the general equilibrium model that relates the rise of fixed-cost intangible inputs to the trends in productivity growth, business dynamism and market power.

3.1. Preferences and Market Structure

A continuum of identical households with unit mass choose the path of consumption that maximizes the following utility function:

$$U = \int_0^\infty \exp(-\rho \cdot t) \cdot \ln C_t \, dt, \tag{4}$$

where C_t is consumption and ρ is the discount factor.[16] Time is continuous and indexed by t, which is suppressed when convenient. The household is endowed with a single unit of labor, which it supplies inelastically.[17] The consumption good is composed of a continuum of intermediate goods, indexed by j. Each good can be produced by the set of firms I_{jt} that own the production technology, a patent, to produce good j at a level of quality $q_{ij} \geq 0$. Quality determines the value that each unit of a good produced by a firm $i \in I_{jt}$ contributes to aggregate consumption. The intermediate goods are competitively aggregated with the following Cobb-Douglas technology:

$$Y = \exp \int_0^1 \ln \left(\sum_{i \in I_j} q_{ij} \cdot y_{ij} \right) dj,$$

[16]It is straightforward to generalize the setup to feature a CRRA utility function. This would change the Euler equation — and the relationship between discount factor ρ and interest rate r — and hence require a different calibration of ρ.

[17]Results, available upon request, are similar if a disutility of labor causes labor supply to be endogenous.

where Y denotes aggregate output, and $y_{ij} \geq 0$ is the amount of good j that is produced by firm i. All output is consumed such that $Y = C$.[18]

The firms that own the patent to produce good j compete à la Bertrand. This implies that, while multiple firms own the patent to produce good j at some level of quality, only one firm will produce the good in equilibrium. In a model where firms have identical production technologies, this would always be the firm with the state-of-the-art patent that allows the firm to produce j at the highest quality level. In this paper's setup, intangibles create heterogeneity in production efficiency. It is optimal for the profit-maximizing aggregator to only demand good j from the firm that offers the highest combination of output and quality $(q_{ij} \cdot y_{ij})$ at a given expenditure. In other words, goods will be produced by the firm that is able to offer the lowest quality-adjusted price p_{ij}/q_{ij}.

3.2. Firms and Intangibles

There is a continuum of firms, indexed by i. In the spirit of Klette and Kortum (2004), firms are able to produce all goods for which they have a patent in their portfolio $J_{it} = \{q_{ij} : j \in \text{patents owned by } i\}$. Given the market structure, firms produce the set of goods $\tilde{J}_{it} \in J_{it}$ for which they are able to offer the lowest quality-adjusted price p_{ij}/q_{ij}.

Following the general setup in Section 2, firms choose the optimal fraction $s_{ij} \in [0,1)$ by which they reduce their marginal costs through the use of intangibles. Firms optimize this fraction separately for each good and choose s_{ij} before production occurs each period. To preserve tractability, the only tangible input is production labor, such that intangibles allow firms to cut the amount of labor required to produce an additional unit of output. The production function reads

$$y_{ij} = \frac{1}{1-s_{ij}} \cdot l_{ij}, \tag{5}$$

where l_{ij} denotes production labor dedicated by i to good j.[19] The marginal cost of producing j equals $mc_{ij} = (1-s_{ij}) \cdot w$, where w is the wage rate. The use of intangibles comes at a cost $f(s_{ij}, \phi_i)$, which satisfies the properties from Section 2: fixed costs increase exponentially in s_{ij}, firms that do not reduce their marginal costs pay no fixed costs, and the costs of reducing marginal costs fully $(s_{ij} \to 1)$ are infinite. To allow a quantification of the model, I choose the following functional form:

$$f(s_{ij}, \phi_i) = (1-\phi_i) \cdot \left(\left[\frac{1}{1-s_{ij}} \right]^{\psi} - 1 \right), \tag{6}$$

[18]The Cobb-Douglas aggregator implies that the demand function has a unit elasticity such that prices of producers in the Bertrand-Nash equilibrium are bound by the marginal cost of the second-best firm. A generalization to CES would imply a similar bound on prices, up to the point that the wedge in marginal costs between the first- and second-best firms exceeds the monopolist markup (see, e.g., Lentz and Mortensen 2008). This gives rise to a kink in the profit function and puts a ceiling on the model's predicted markups. Given the absence of such a ceiling on markups in the data and to preserve tractability, I instead rely on the Cobb-Douglas technology.

[19]In independent work, Korinek and Ng (2019) also model digitization as a shift from marginal to fixed costs. Their model features heterogeneity in the maximum fraction of marginal costs that firms are able to cut.

where ψ is a curvature parameter and ϕ_i captures the efficiency with which firms are able to implement intangible technologies. Firms draw their type ϕ_i from a known discrete distribution $G(\phi)$ at birth and benefit from their level of intangible efficiency on each good that they produce. Note that fixed costs are not sunk, as firms pay the fixed costs at each time t. The motivation for that is twofold. First, Li and Hall (2020) estimate depreciation rates of software investments to range between 30 and 40% per year. This implies that firms must spend considerable amounts each year to maintain a constant level of software. Second, an increasing share of enterprise software is sold *as a service* (SaaS), where firms pay periodic fees instead of an upfront cost for perpetual use.[20] Note that firms also accumulate intangible capital in the spirit of Corrado et al. (2009): firms invest in research and development, which persistently affects both firm size and national income.

3.3. Innovation

3.3.1. Research and Development

Firms expand their portfolio of patents by investing in research and development (R&D). When investing, firms choose the Poisson flow rate $x_i \geq 0$ with which a new patent is added to their portfolio. In exchange for achieving x_i, firms employ rd^x researchers along

$$rd^x(x_i) = \eta^x \cdot x_i^{\psi^x} \cdot n_i^{-\sigma}, \tag{7}$$

where $\psi^x > 1$ and $0 \leq \sigma \leq \psi^x - 1$. The number of researchers that the firm employs is convex in the rate of innovation and declines in the number of goods that the firm produces, n_i. The former implies that the marginal return to R&D is diminishing within each time t. The latter is an assumption from Klette and Kortum (2004), and reflects the assumption that large firms have more in-house knowledge or organizational capital than small firms. Practically, the presence of $n_i^{-\sigma}$ governs the relationship between firm size and firm growth. For $\sigma = \psi^x - 1$, the model satisfies Gibrat's law of constant firm growth in size, while for $\sigma = 0$ a firm's growth declines rapidly with size. Following Akcigit and Kerr (2018), I allow for an intermediate case between these two extremes, and estimate $\sigma \in [0, \psi^x - 1]$ by targeting the empirical relationship between size and growth in the data.

A firm that innovates successfully becomes the owner of a state-of-the-art patent for a random good j. Innovation is not directed, in the sense that firms are equally likely to innovate on all products. As in Aghion and Howitt (1992), the state-of-the-art patent allows firm i to produce its new good at a quality level that is a multiple $(1+\lambda_{ij})$ of the level of the current producer of the good:

$$q_{ij} = q_{-ij} \cdot (1 + \lambda_{ij}),$$

where $-i$ denotes the incumbent of good j while λ_{ij} denotes the realized innovation step size, which is drawn from an exponential distribution with mean $\bar{\lambda}$:

$$\lambda \sim \text{Exp}(\bar{\lambda}).$$

[20] For example, 35% of Microsoft's enterprise sales in Q2 of 2019 came from SaaS, at an annual growth of 48%.

3.3.2. Innovation and Intangibles

Innovation in the model is different from the standard Klette and Kortum (2004) setup because the innovator of a certain good will not necessarily become its new producer. Innovators always become the producer in other models because firms have identical marginal costs while the innovator owns the patent to produce at the highest quality level. Here, the owner of a lower-quality patent may still be the sole producer if it can offer the best combination of quality and price. The lowest prices that the incumbent and the innovator are willing to set are their respective choke prices. The choke price $p^{choke}(\phi_i)$ is the price at which, after payment of the fixed costs, firm profits are zero.[21] If the incumbent has a lower choke price than the innovator does, the incumbent can undercut the innovator on price if the quality of the innovator is sufficiently close to that of the incumbent. Formally, innovator i only becomes the new producer of a good j that is initially produced by -i if

$$\frac{q_{ij}}{p^{choke}(\phi_i)} \geq \frac{q_{-ij}}{p^{choke}(\phi_{-i})},$$

where the choke price is a decreasing function of ϕ_i because high-ϕ are able to reduce their marginal costs by a greater fraction for a given expenditure on intangibles. Rewriting yields

$$\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1, \tag{8}$$

The innovator is able to offer product j at a superior quality to that of the incumbent, but the incumbent can hold on to its product if it has a sufficiently low choke price. A greater difference between the choke prices is needed when the innovator has drawn a significant innovation (the realization of λ_{ij} is high). The innovator will always become the new producer if its ϕ_i is the same or higher than that of the incumbent.

3.3.3. Quality and Intangibles

It is useful to highlight the difference between quality and price in the model. In most models of growth through creative destruction, the two are isomorphic. Prices reflect the ability of firms to produce at low marginal costs (that is; with high productivity). It may seem that this is equivalent to quality, in the sense that a firm can increase its effective output $q_{ij} \cdot y_{ij}$ using the same quantity of tangible inputs by either selling at higher quality or by using a greater amount of intangibles.

The difference between the two lies in their contribution to long-term growth. Innovation raises the state-of-the-art quality with which good j can be produced. If an innovating firm successfully takes over production, this offers both a private- and an economy-wide benefit. The private benefit is the stream of profit that the firm earns while it produces j. The economy-wide benefit is that all future innovations on j are step-wise improvements over q_{ij}: the innovation by firm i al-

[21] This is with slight abuse of notation, as $p^{choke}(\phi_i)$ also depends on output Y and wage w. It is expressed only in terms of ϕ_i because the ratio of choke prices between any two firms only depends on their relative ϕs.

lows good j to be produced at a permanently higher level of quality. This positive externality makes the step-wise improvement of quality across products the source of long-term economic growth.

Intangibles do not come with a similar externality. They only improve production efficiency for the current producer. Intuitively, the fact that the incumbent is efficient at using software applications to reduce marginal costs does not benefit an innovating firm when it takes over production.

3.4. Entry and Exit

There is a mass of entrepreneurs that invest in R&D to obtain patents to produce goods that are currently owned by incumbents. The R&D cost function is analogous to the cost function for innovation by incumbents:

$$rd^e(e) = \eta^e \cdot e^{\psi^e}, \tag{9}$$

where $rd^e(e)$ denotes the number of researchers employed by potential entrants to achieve start-up rate e, and where $\eta^e > 0$, $\psi^e > 1$. Entrepreneurs that draw an innovation improve the quality of a random good that is currently produced by an incumbent. In similar spirit to models where firms draw idiosyncratic productivities at birth (e.g. Hopenhayn 1992, Melitz 2003), entrants then draw their intangible productivity $\phi_e \in \Phi$ from the known distribution $G(\phi)$, and learn about their incumbent's intangible efficiency. The entrant becomes the new producer if it has drawn a sufficiently large step-size λ_{ej} to overcome any difference in choke prices along condition (8).

A firm exits the economy if it does not produce any good in its patent portfolio J_i. This happens when entrants or other incumbent develop higher-quality versions of the sole good that a firm produces, as explained in the next section.

3.5. Creative Destruction

Firms cease to produce a good if a different incumbent or an entrant successfully innovate on that product. The rate at which this happens is the rate of creative destruction, $\tau(\phi_i)$. The rate of creative destruction is endogenous, as it is determined by the respective efforts that incumbents and entrants put into innovation. It is a function of the firm's intangible efficiency ϕ_i, because a firm with a relatively high intangible efficiency is more likely to be able to undercut an innovative challenger on price. The rate of creative destruction for a firm with efficiency ϕ_i is given by

$$\tau(\phi_i) = \sum_{\phi_h \in \Phi} \text{Prob}\left(\lambda_{ih} \geq \frac{p^{choke}(\phi_h)}{p^{choke}(\phi_i)} - 1\right) \cdot \left[\sum_{n=1}^{\infty} M(\phi_h, n) \cdot x(\phi_h, n) + e \cdot G(\phi_h)\right], \tag{10}$$

where $M(\phi_h, n)$ denotes the measure of firms with intangible efficiency ϕ_h that produce n products. The outer-summation reflects that an incumbent with intangible efficiency ϕ_i faces innovative competitors from each intangible-efficiency level $\phi_h \in \Phi$. Within the summation there are two terms: the probability that an innovation by a firm with efficiency ϕ_h is successful, multiplied by innovation efforts by firms with that level of efficiency. Under the exponential distribution, the probability that condition (8) is satisfied when i is the incumbent and h is the innovator equals

$$\text{Prob}\left(\lambda_{hj} \geq \frac{p^{choke}(\phi_h)}{p^{choke}(\phi_i)} - 1\right) = \bar{\lambda}^{-1}\exp\left(-\bar{\lambda}^{-1}\cdot\left[\frac{p^{choke}(\phi_h)}{p^{choke}(\phi_i)} - 1\right]\right), \tag{11}$$

where the right-hand side is the cumulative density function of the exponential distribution with mean $\bar{\lambda}$. This probability is strictly lower when the incumbent is a high-ϕ firm, as these have a lower choke price. The term for innovation effort in (10) contains two parts. The first captures innovation by incumbents of type ϕ_h. As is shown below, a firm's innovation effort is a function of its intangible efficiency as well as the number of products it produces, which explains the inclusion of the summation over n. The Poisson rate $x(\phi_h, n)$ is multiplied by the measure $M(\phi_h, n)$ to obtain total innovation effort. The second term measures innovation by entrants of type ϕ_h. It is the product of the entry rate e and the probability $G(\phi_h)$ that the entrant is of type ϕ_h.

3.6. Optimal Pricing and Intangibles

Firms choose their optimal price p_{ij} and marginal costs $(1 - s_{ij}) \cdot w$ to statically maximize profits. The optimal price is determined by the efficiency wedge between the firm that produces good j and the efficiency of the second-best firm for that good. The following timing assumption applies. At the start of each time t, all firms with a patent to produce good j observe the qualities and intangible efficiencies of all firms with a patent to produce good j. They then choose s_{ij} and commit to paying the associated fixed costs $f(s_{ij}, \phi_i)$ and subsequently post their prices and produce the goods demanded by consumers. In the Nash equilibrium of the associated simultaneous move game, firms that are unable to offer the lowest quality-adjusted price have no incentive to set $s_{ij} > 0$. Their marginal cost therefore equals the wage w. The demand for output from the firm with the lowest quality-adjusted choke price has a unit demand elasticity: $y_{ij} = Y \cdot p_{ij}^{-1}$. The profit-maximizing price of the firm i with the lowest quality-adjusted choke price is therefore bound by the marginal cost of the firm with the second-lowest choke price -i, adjusted for differences in quality:

$$p_{ij} = mc_{-ij} \cdot \frac{q_{ij}}{q_{-ij}},$$

where $-i$ identifies the second-best firm, $mc_{-ij} = w$, and $q_{ij}/q_{-ij} - 1$ is innovation realization λ_{ij}. The markup μ_{ij} of firm i is found by dividing the profit-maximizing price by firm i's marginal cost $w \cdot (1 - s_{ij})$ and by inserting the innovation step-size λ_{ij} for the ratio of qualities:

$$\mu_{ij} = \frac{1 + \lambda_{ij}}{1 - s_{ij}}, \tag{12}$$

which yields that markups increase in the difference in quality between the producer and the second-best firm, as well as the firm's use of intangibles. Note that while intangibles increase the markup, profits do not increase proportionally because the firm incurs an expense on intangibles. A part of the increase in markups is therefore a compensation for fixed costs.

To find the optimal intangible fraction s_{ij}, consider the definition of operating profits:

$$\pi_{ij} = (p_{ij} - mc_{ij}) \cdot y_{ij} - w \cdot f(s_{ij}, \phi_i),$$

where the fixed-cost function (6) is multiplied by w, as costs are denominated in terms of labor. Inserting the demand function and markups (12) gives the following first-order condition:

$$s_{ij} = 1 - \left([1 + \lambda_{ij}] \cdot \frac{w}{Y} \cdot \psi \cdot (1 - \phi_i) \right)^{\frac{1}{\psi+1}}, \tag{13}$$

or $s_{ij} = 0$ when the right-hand side is negative. It follows that firms with higher intangible efficiencies are able to reduce their marginal costs by a greater fraction and consequently have higher markups. Note that the firm with the lowest quality-adjusted choke price sets s_{ij} along (13) irrespective of the second-best firm's ϕ, because that firm always sets $s_{-ij} = 0$ in the Nash equilibrium.

3.7. Equilibrium

I now characterize the stationary equilibrium where productivity, output and wages grow at rate g.

3.7.1. Optimal Innovation Decisions

Firms choose the level of spending on research and development that maximizes firm value. The associated value function, where notation is borrowed from Akcigit and Kerr (2018), reads as

$$rV_t(\phi_i, \tilde{J}_i) - \dot{V}_t(\phi_i, \tilde{J}_i) = \max_{x_i} \left\{ \begin{array}{c} \sum_{j \in \tilde{J}_i} \pi_t(\phi_i, \lambda_{ij}) + \tau(\phi_i) \cdot \left[V_t(\phi_i, \tilde{J}_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i, \tilde{J}_i) \right] \\ + x_i \cdot \text{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \mathbb{E}_{\phi_i} \left[V_t(\phi_i, \tilde{J}_i \cup_+ \lambda_{ij}) - V_t(\phi_i, \tilde{J}_i) \right] \\ - w_t \eta_x (x_i)^{\psi_x} n_i^{-\sigma} - F(\phi_i, n_i) \end{array} \right\}$$

The first line on the right-hand side contains the sum of all good-specific items. It is the sum of contemporaneous profits for a firm that sets prices along (12) and intangibles along (13), and the change in firm value if the firm would cease production of good j because of creative destruction by entrants or other incumbents. $V_t(\phi_i, \tilde{J}_i \setminus \{\lambda_{ij}\})$ denotes the value of producing the set of goods $\tilde{J}_i$ except some good j with innovation realization λ_{ij}. The bottom two lines are not specific to goods. The first line gives the expected increase in firm value from external innovation. $V(\phi_i, \tilde{J}_i \cup_+ \lambda_{ij})$ denotes the firm's value if it successfully takes product j from firm $-i$. The change in firm value is multiplied by the innovation rate and the probability that the firm is able to offer a sufficiently low quality-adjusted price. The final line gives the costs of R&D and a fixed term $F(\phi_i, n_i)$. Firms must pay the latter in order to operate, and it is assumed to equal the option value of research and development. This ad-hoc restriction, borrowed from Akcigit and Kerr (2018), ensures that the value function is linear in the number of goods that firms produce, such that the model admits an analytical first-order condition. In Section 6 I remove this assumption and show that, though significantly reducing tractability, the results are qualitatively and quantitatively robust.

Proposition 1. *The value function of a firm with intangible efficiency ϕ_i that produces a portfolio of goods $\tilde{J}_i$ with cardinality n_i grows at rate g along the balanced growth path and is given by*

$$V(\phi_i, \tilde{J}_i) = \sum_{j \in \tilde{J}_i} \pi(\phi_i, \lambda_{ij}) \cdot (r - g + \tau(\phi_i))^{-1},$$

which is increasing in ϕ_i. The optimal rate of of innovation reads as

$$x(\phi_i, n_i) = \left(\text{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \mathbb{E}_{\phi_i} \left[\frac{\pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)} \right] \cdot (\eta^x \cdot \psi^x \cdot w_t)^{-1} \right)^{\frac{1}{\psi^x - 1}} \cdot n_i^{\frac{\sigma}{\psi^x - 1}}. \quad (14)$$

The optimal entry rate is given by

$$e = \left(\sum_{\phi_e \in \Phi} G(\phi_e) \cdot \text{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_h)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \mathbb{E}_{\phi_e} \left[\frac{\pi(\phi_e, \lambda_{ij})}{r - g + \tau(\phi_e)} \right] \cdot (\eta^e \psi^e w_t)^{-1} \right)^{\frac{1}{\psi^e - 1}}. \quad (15)$$

Proof: Appendix A.

First-order condition (14) is intuitive. Firms engage in more innovation when the expected increase in value is higher, and invest less when the innovation cost-parameters are high. Innovation increases in the firm-size n_i — although if $\sigma < \psi^x - 1$, the firm's expected growth rate will decline with size. Firms with a higher intangible efficiency ϕ_i choose a higher innovation rate because their ability to reduce marginal costs increases profitability. They furthermore face a lower rate of creative destruction, which decreases the effective discount factor. Firms with higher ϕ_is also have a higher probability of successfully becoming the new producer on products that they innovate on. Jointly, these effects cause a positive relationship between ϕ_i and the rate of innovation.

Innovation by entrants (15) is such that the marginal cost of increasing the entry rate e is equal to the expected value of producing a single good, adjusted for the probability that the entrant is able to take over production from the incumbent by offering a sufficiently low quality-adjusted price. Because entrants only learn about their type after they have drawn an innovation, the expectation of the value of producing a good is taken over the distribution of firm types at entry $G(\phi)$.

3.7.2. Intangibles and Growth: Mechanism and Evidence

Equation (14) implies a positive relationship between ϕ_i and a firm's innovation efforts. In Appendix G I show that, in line with the model, there is a significantly positive within- and between-firm correlation between fixed costs and R&D expenditures in the data from Section 2. These firms also grow significantly faster. How is this consistent with a slowdown of productivity growth?

A homogeneous increase of ϕ_i improves profitability for all firms and therefore raises innovation rates and productivity growth. That is not the case, however, when only a fraction of firms receive a higher intangible efficiency. High-ϕ_i firms would have a greater incentive to invest in research and development, which leads them to produce a disproportionate fraction of all goods. This has two negative externalities. First, the incentives to engage in R&D for lower-ϕ_i firms decline, as some of their innovations are now unsuccessful. Second, there is a decline in the rate of entry; because high-ϕ_i firms expand, it is more likely that entrants face a high-ϕ_i incumbent than that they, themselves, are high-ϕ_i firms. In Sections 5 and 6, I show that these externalities undo the positive effect of the high innovation rates by high-ϕ_i firms for plausible calibrations. Indeed, the increase in R&D by high-ϕ_i firms can be so large that aggregate R&D increases (in line with Figure 1), but growth declines because R&D concentrates among a measure number of firms.

3.7.3. Dynamic Optimization by Households

Maximizing life-time utility with respect to consumption and savings subject to the budget constraint gives the usual Euler equation,

$$\frac{\dot{C}}{C} = r - \rho, \tag{16}$$

combined with the transversality condition. Along the balanced growth path, consumption grows at the same rate as output and productivity, such that $r - g = \rho$.

3.7.4. Firm Measure and Size Distribution

The optimal innovation rate in (14) is a function of a firm's intangible input efficiency ϕ_i and the number of goods n_i it produces. The rate of creative destruction (and hence the growth rate of output and productivity) therefore depends on the equilibrium distribution of n and ϕ across firms. Along the balanced growth path, these distributions are stationary. To find the stationary distributions, consider the law of motion for the measure of firms that produce more than one product:

$$\begin{aligned}\dot{M}(\phi_i, n) &= \big(M(\phi_i, n-1)\cdot x(\phi_i, n-1) - M(\phi_i, n)\cdot x(\phi_i, n)\big)\cdot \\ &\text{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1\right) + \big(M(\phi_i, n+1)\cdot[n+1] - M(\phi_i, n)\cdot n\big)\cdot \tau(\phi_i),\end{aligned} \tag{17}$$

where the first term captures entry into and exit out of measure $M(\phi_i, n)$ through innovation by firms of type ϕ_i with $n-1$ products and n products, respectively. The second term captures entry and exit of firms with $n+1$ and n products that ceased producing one of their products through creative destruction. For the measure of single-product firms, the law of motion reads as

$$\dot{M}(\phi_i, 1) = \big(e\cdot G(\phi_i) - x(\phi_i, 1)\cdot M(\phi_i, 1)\big)\cdot\text{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1\right) + \big(2\cdot M(\phi_i, 2) - M(\phi_i, 1)\big)\cdot\tau(\phi_i). \tag{18}$$

The stationary firm-size distribution follows from setting both equations to zero for each n. The fraction of goods that is produced by firms with intangible efficiency ϕ_i is given by

$$K(\phi_i) = \frac{\sum_{n=1}^{\infty} n \cdot M(\phi_i, n)}{\sum_{\phi_h \in \Phi} \sum_{n=1}^{\infty} n \cdot M(\phi_h, n)}. \tag{19}$$

3.7.5. Labor Market Equilibrium

The solutions to the static and dynamic optimization problems of firms allow the labor market equilibrium conditions to be defined. Labor is supplied inelastically by households at a measure standardized to 1. Equilibrium on the labor market requires that

$$1 = L^p + L^f + L^{rd} + L^e,$$

where L^p is the labor used to produce intermediate goods. Inserting the unit-elastic demand function, markup (12) and intangible first-order condition (13) into $L^p = \int_0^1 \mathbf{1}_{j \in \tilde{f}_i} l_{ij} di\, dj$ yields

$$L^p = \int_0^1 \int \mathbf{1}_{j \in \tilde{f}_i} \cdot \frac{Y}{w} \cdot \left[1 - \left([1+\lambda_{ij}] \cdot \frac{w}{Y} \cdot \psi \cdot (1-\phi_i) \right)^{\frac{1}{\psi+1}} \right] \cdot (1+\lambda)^{-1}\, di\, dj,$$

where $\mathbf{1}_{j \in \tilde{f}_i}$ is the indicator function that equals one when firm i produces good j. L^f is the labor used to fulfill the intangible fixed costs:

$$L^f = \int_0^1 \int \mathbf{1}_{j \in \tilde{f}_i} \cdot \left[\left([1+\lambda_{ij}] \cdot \frac{w}{Y} \cdot \psi \cdot (1-\phi_i) \right)^{-\frac{\psi}{\psi+1}} - 1 \right] \cdot (1-\phi_i)\, di\, dj.$$

L^{rd} is the labor involved with research and development carried out by existing firms:

$$L^{rd} = \sum_{\phi_i \in \Phi} \sum_{n=1}^{\infty} \left[M_{\phi_i, n} \cdot \eta^x \cdot x(\phi_i, n)^{\psi^x} \right],$$

while L^e is the labor involved with research and development carried out by entrants $L^e = \eta^e \cdot e^{\psi^e}$, where innovation rates $x(\phi_i, n)$ and e are dynamically optimized along (14) and (15).

3.7.6. Aggregate Variables

I can now characterize the economy's aggregate variables. The equilibrium wage is given by

$$w = \exp\left(\int_0^1 \int \mathbf{1}_{j \in \tilde{f}_i} \cdot \ln \left[\frac{q_{ij}}{1-s_{ij}} \right] di\, dj \right) \cdot \exp\left(\int_0^1 \int \mathbf{1}_{j \in \tilde{f}_i} \cdot \ln \left[\frac{1-s_{ij}}{1+\lambda_{ij}} \right] di\, dj \right). \tag{20}$$

The first term of (20) is the standard CES productivity term. The second term is the inverse of the expected markup. Note that a rise in the use of intangibles has no effect on the level of the wage because s_{ij} cancels out. While a firm that deploys more intangibles becomes productive, it is able to proportionally raise its markups. These have offsetting effects on the level of the wage.

Aggregate output is given by

$$Y = L^p \cdot \exp\left(\int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \cdot \ln \left[\frac{q_{ij}}{1 - s_{ij}}\right] di\, dj\right) \cdot \frac{\exp \int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \cdot \ln \mu_{ij}^{-1}\, di\, dj}{\int_0^1 \int \mathbf{1}_{j \in \tilde{J}_i} \cdot \mu_{ij}^{-1}\, di\, dj}. \tag{21}$$

Proof: Appendix A.

As in the model with heterogeneous markups and misallocation by Peters (2019), the last term captures the loss of efficiency due to the dispersion of markups. If all markups are equalized the term is equal to 1, while it declines as the variance of markups increases. Total factor productivity is the product of the second- and the last term in (21).

Equation (21) reveals that a rise in the use of intangibles has two counteractive effects on the level of output. The spread of markups increases when the average s_{ij} increases along (12), because s_{ij} amplifies the heterogeneity in markups caused by the heterogeneous innovation steps (the second term in (21)). On the other hand, the increase in s_{ij} has a direct positive effect on total factor productivity because it increases the CES productivity index (the first term in (21)). As will be clear below, the second effect dominates the first effect in feasible calibrations. That means that a rise in the use of intangibles initially has a positive effect on the level of output and on total factor productivity. The next proposition shows, however, that this may not be the case for growth.

3.7.7. Growth

The growth rate of total factor productivity and output is a function of creative destruction.

Proposition 2. *The constant growth rate of total factor productivity, consumption C, aggregate output Y and wages w is given by*

$$g = \sum_{\phi_i \in \Phi} K(\phi_i) \cdot \tau(\phi_i) \cdot \mathbb{E}_{-\phi_i}(\lambda_{hj}), \tag{22}$$

where $\mathbb{E}_{-\phi_i}(\lambda_{hj})$ is the expected realization of λ_{hj} when a firm with ϕ_i is the incumbent on a product line before a different firm h becomes the new producer due to successful innovation.

Proof: Appendix A.

The proposition states that growth equals the product of the expected increase in quality if a good gets a new producer and the rate at which this happens, weighted by the fraction of product lines that firms of each intangible efficiency own.

Equation (22) shows the counteracting effects of an increase in ϕ at a subset of firms. On the one hand, firms with a higher ϕ have a greater incentive to invest in research and development, which causes the rate of creative destruction to increase. On the other hand, even at a constant innovation rate, the presence of high-ϕ firms has a negative effect on the rate of creative destruction because firms with lower productivities ϕ have a lower probability of successfully becoming the new producer. This has not only a direct effect on growth at given innovation rates, but also an indirect effect, as these firms reduce their expenditure on research and development.

3.7.8. Equilibrium Definition

Definition 1. *The economy is in a balanced growth path equilibrium if for every t and for every intangible productivity $\phi_i \in \Phi$, the variables $\{r, e, L^p, g\}$ and functions $\{x(n_i, \phi_i), K_{\phi_i}, M_{\phi_i}, s(\phi_i, \lambda_{ij}), \tau(\phi_i)\}$ are constant, $\{Y, C, w, Q\}$ grow at a constant rate g that satisfies (22), aggregate output Y satisfies (21), innovation rates $x(n_i, \phi_i)$ satisfy (14), the entry rate e satisfies (15), firm distribution K_{ϕ_i} and measure M_{ϕ_i} are constant and satisfy (17) and (18), markups $\mu(\phi_i, \lambda_{ij})$ satisfy (12), the fraction of marginal costs reduced through intangibles $s(\phi_i, \lambda_{ij})$ satisfies (13) for all λ_{ij}, the rate of creative destruction $\tau(\phi_i)$ satisfies (10), and both the goods and labor market are in equilibrium such that $Y = C$ and $L^p = 1 - L^s + L^{rd} + L^e$.*

4. Quantification

This section outlines how the model is quantified. I first discuss the calibration and structural estimation strategy, and then discuss the extent to which the model is able to replicate a set of targeted and untargeted moments along the original balanced growth path.

4.1. Calibration

In the baseline calibration all firms have the same intangible efficiency ϕ, which leaves nine parameters to be calibrated. Five parameters are calibrated using a structural estimation, while four others are taken from the literature. The structural estimation is conducted separately for the United States and France, using the micro data from Section 2.

4.1.1. Externally Calibrated Parameters

The model is calibrated at an annual frequency. I calibrate the curvature of R&D for entrants (ψ^e) and incumbents (ψ^x) to 2. This is a key parameter because it determines the concavity of the return to R&D. If innovative activities concentrate among fewer firms, the fact that $\psi^x > 1$ implies that the average effect of these investments on growth is lower. The literature that studies the elasticity of R&D with respect to the user costs ($\epsilon_{x,w}$) of such activities finds elasticities around -1.0 for tax credit changes (see, e.g. Bloom et al. 2002 for a review).[22] The parameter ψ^x is related to $\epsilon_{x,w}$ along

$$\psi^x = -\frac{\epsilon_{x,w} - 1}{\epsilon_{x,w}},$$

and is therefore set to 2. The same value is used for corresponding parameters in Akcigit and Kerr (2018) and Acemoglu et al. (2018).

[22] A recent large-scale analysis from cross-country micro data by Appelt et al. (2020) finds an average cost elasticity of -0.66. The corresponding ψ^x is 2.53, which yields a greater reduction in productivity growth.

I calibrate the curvature parameter ψ of fixed cost function $f(\cdot)$ to match empirical estimates of the pass-through of marginal costs to markups. To see how these are related, note that the first-order conditions for markups (12) and for intangibles (13) imply an equilibrium log markup of

$$\ln \mu_{ijt} = \ln(1+\lambda_{ij}) - \ln\left((1+\lambda_{ij}) \cdot \frac{w_t}{Y_t} \cdot \psi \cdot (1-\phi_i)\right) \cdot \frac{1}{\psi+1}.$$

The elasticity of marginal costs with respect to wages is $(\psi+1)/(\psi+2)$, such that the elasticity of markups with respect to marginal costs at a given level of Y is $-(\psi+2)^{-1}$. I set ψ to 2, which achieves a pass-through of -25%. Empirical estimates of this elasticity vary. Amiti et al. (2019) find a pass-through of -35% in their main results. In robustness checks on the full sample they find values between -39% and -25%. For firms with fewer than 100 employees they find coefficients of -3%. Table A4 in Appendix F shows that the results are robust to $\psi = 0.86$, yielding a -35% pass-through.

The discount rate ρ is set to 0.01, which gives rise to a 2.3% risk-free rate.

4.1.2. Structurally Estimated Parameters

The remaining five parameters are estimated using indirect inference by matching moments from either the U.S. Compustat data on listed firms or the French administrative data. The U.S. calibration targets moments for 1980, which is the first year that firm variables from Compustat can be complemented by administrative data on business dynamism. The French calibration targets moments in the first year of the data (1994), or the first available year for surveys.

I use the Genetic Algorithm to choose combinations of parameters within broad bounds on their possible values. For the given parameterization I solve the model as a fixed point using the algorithm described in Appendix E. Using the equilibrium values for innovation and entry rates, the firm-size distribution, rates of creative destruction and aggregate quantities such as the efficiency wedge, wages and output, I simulate the economy for 32,000 firms until the distribution of s_{ij} has converged, and simulate data for five more years to collect moments on the simulated sample.[23] The Genetic Algorithm then updates the combinations of parameters based on a comparison of the theoretical and data moments along the following objective function:[24]

$$\min \sum_{k=1}^{5} \frac{|\,\text{model}_k - \text{data}_k\,|}{(|\,\text{model}_k\,| + |\,\text{data}_k\,|)\cdot 0.5} \cdot \Omega_k, \tag{23}$$

where model_i and data_i respectively refer to the simulation and data for moment i with weight Ω_i.

The following moments are used for the U.S. calibration. I calibrate the initially homogeneous intangible efficiency parameter ϕ to match the 1980 ratio of fixed to variable costs of 13.9% in Section 2. The cost scalar of R&D by entrants (η^e) is estimated by targeting the entry rate of 13.8% for 1980 in the Business Dynamics Statistics. The cost scalar of innovation by existing firms (η^x) is

[23] The firm simulation builds computationally on Akcigit and Kerr (2018) and Acemoglu et al. (2018).

[24] The Genetic Algorithm is a method to find global minimums that is inspired by the process of natural selection. It involves taking convex combinations (children) of parameter vectors (parents). The performance of children on the optimization criteria determines their likelihood of becoming parents in the next generation of the algorithm. The algorithm was significantly better at finding global minimums than alternatives such as Simulated Annealing.

Table 4: Overview of Parameters

Parameter	Description	Method	Value (U.S.)	Value (France)
ρ	Discount rate	External	.010	.010
ψ	Intangibles cost elasticity	External	2.00	2.00
ψ^x	Cost elasticity of innovation (incumbents)	External	2.00	2.00
ψ^e	Cost elasticity of innovation (entrants)	External	2.00	2.00
η^x	Cost scalar of innovation (incumbents)	Indirect inference	5.52	2.15
η^e	Cost scalar of innovation (entrants)	Indirect inference	2.98	3.27
λ	Average innovation step size	Indirect inference	.065	.067
σ	Relationship firm-size and firm-growth	Indirect inference	.475	.600
ϕ	Intangible efficiency	Indirect inference	.806	.737

estimated by targeting the average ratio of R&D over sales for firms with positive expenditures in 1980, at 2.5%. Following Akcigit and Kerr (2018), I calibrate the parameter that governs the extent to which R&D scales with size (σ) by targeting an OLS regression of size on growth along

$$\Delta_i (p \cdot y) = \alpha_s + \beta \cdot \ln (p_i \cdot y_i) + \varepsilon_i, \tag{24}$$

where the left-hand side is the growth rate of sales using the measure of growth in Davis et al. (2006) while α_s is a sector fixed effect. Akcigit and Kerr (2018) run this regression on Census data and find a β of -0.035, which implies that a firm with 1% greater sales is expected to grow 0.035% less. I target a growth rate of productivity along the balanced growth path of 1.3%, which is the average growth rate of total factor productivity between 1969 and 1980 in the Fernald series.

The calibration for France relies on the French counterparts of the U.S. moments. The intangible efficiency parameter ϕ is calibrated by matching the 1994 ratio of fixed to variable costs of 9.5% in Section 2. The cost scalar of research and development by entrants (η^e) is estimated by targeting an entry rate of 10%. This is the fraction of firms that enter the FARE-FICUS dataset for the first time in 1995, the second year for which data is available and therefore the first year that entry is observed. The cost scalar of innovation by existing firms (η^x) is estimated by targeting the average ratio of R&D over sales in the CIS for 1996, which is 3.1%. I calibrate σ to match the coefficient β in (24) using data on French firms for 1994-1995. The estimated β is -0.035, coincidentally the same coefficient as for the U.S. I target a productivity growth rate of 1.3%, which is the average growth rate of total factor productivity between 1969 and 1994 in the Penn World Tables.

Table 4 presents an overview of the calibrated and estimated parameters. The lower R&D intensity of U.S. firms gives rise to a higher innovation-cost scalar η^x, while the higher ratio of fixed-to variable costs of U.S. firms causes their baseline estimated intangible efficiency ϕ to be higher than that of the French firms. The estimated innovation-step size is similar for both countries.

4.2. Model Properties

A comparison of theoretical and empirical targeted moments is provided in Table 5. The first column lists the parameter that corresponds most closely to the moment, the second column describes the moment, and the third column summarizes the moment's weight in the structural es-

Table 5: Comparison of Theory and Data for Targeted Moments

			United States		France	
Parameter	**Moment**	**Weight Ω**	**Model**	**Data**	**Model**	**Data**
λ	Long-term growth rate of productivity	1	1.3%	1.3%	1.3%	1.3%
ϕ	Fixed costs as a fraction of total costs	2	14.2%	13.9%	9.5%	10.3%
σ	Relation between firm growth and size	1	-.035	-.035	-.035	-.035
η^e	Entry rate (fraction of firms age 1 or less)	1	13.5%	13.8%	10.6%	8.6%
η^x	Ratio of research and development to sales	1	2.4%	2.5%	2.9%	3.2%

Notes: Data columns present the empirical moments while model columns present the theoretical moments. U.S. moments are for 1980 except for the regression coefficient of firm growth on firm size, which is taken from Akcigit and Kerr (2018). French moments are for 1994 or the first subsequent year for which the moment is present in the micro data.

timation. All moments receive the same weight except the share of fixed costs, which is assigned a weight of two. The model is able to match moments on growth and the relationship between firm growth and firm size precisely for both countries. R&D intensities and fixed costs are also matched closely, while the estimated model underestimates entry in France by two percentage points.

The firm-size distribution is untargeted. The Cobb-Douglas aggregator implies that a firm's revenue is determined by the number of goods that it produces, which is plotted against data in Figure 3. I rely on the Compustat Segments data for the U.S. to count the number of NAICS industries that firms operate in (Figure 3a).[25] This is the orange-circled line. Results show that U.S. listed firms operate in more sectors than the model predicts. Note that the Compustat segments are an imperfect measure of the number of products that firms produce because firms apply heterogeneous reporting standards on what a segment is. Further, 29.5% of firms do not report their segments at all. The green-squared line plots an alternative distribution of the product count, setting the number of products to one for non-reporting firms. This brings the distribution closer to what is predicted. The difference between the fraction of firms with 2 and 3 (and 3 and 4) products is also accurately predicted. Figure 3b plots the same results for France. Data come from the *Enquête Annuelle de Production dans L'Industrie* (EAP). Although this dataset is available only for firms in manufacturing, it does contain identifiers for each product that the firm sells.[26] The figure shows that the distribution of the number of products that firms sell is closely matched.

Table A3 in Appendix F presents a set of additional untargeted moments. The left-hand columns present moments from the U.S. data while the right-hand columns present moments from France. The first panel analyzes the relationship between size and age. Size is measured as sector-deflated sales, while age is measured as years since creation in France and as years since entry into Compustat for the U.S. Both are transformed to within-year quartiles indexed from 1 to 4.[27] For the U.S., the model accurately predicts that small firms are more likely to exit and less likely to stop producing a product, but cannot explain the relationship between exit and age. This could be because U.S. exits are calculated within Compustat, which can reflect that a firm was acquired or delisted. Exit rates for the U.S. are therefore not necessarily due to firm closure. The model correctly predicts that young firms are on average smaller than older firms, as they have had less time to accumulate

[25] The first year with NAICS segment codes is 1990, which is plotted here. Details are provided in Data Appendix B.

[26] The first year of the survey is 2009, which is plotted here. Details are provided in Data Appendix B.

[27] E.g. the first entry implies that firms in age quartile 1 have a 1.21 average score on a 1-4 scale of the size quartiles.

patents through R&D. The model also correctly predicts for France that young and small firms are more likely to exit and less likely to stop producing one of their products.

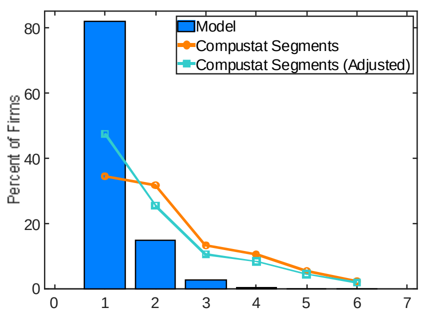
5. Analysis

This section contains the main exercise: a quantitative analysis of the effect of a rise of intangibles on productivity growth, business dynamism and markups. I first outline how high-intangible firms are introduced in Section 5.1, and analyze how they change the balanced growth path in Section 5.2. The transition path from the old to the new steady state is presented in Sections 5.3 and 5.4.

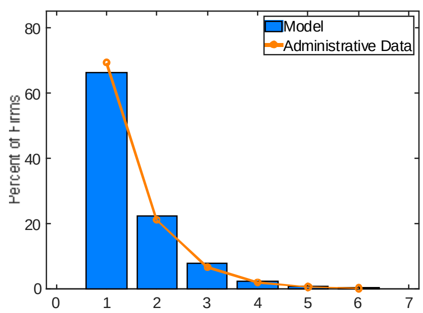
5.1. Introducing Heterogeneous Intangible Efficiency

To model the rise of intangibles, I introduce a group of firms with an intangible efficiency ϕ_j that exceeds the homogeneous intangible efficiency in the initial calibration of Section 4.[28] Two parameters characterize the introduction of high-intangible firms: their level of intangible efficiency, $\overline{\phi}$, and the fraction of entrants that receive it, $G(\overline{\phi})$. I calibrate $\overline{\phi}$ by targeting the increase in the ratio of software investments over private sector value added. This corresponds directly to $\overline{\phi}$ because a higher average intangible efficiency leads to a greater use of intangibles. For the U.S. calibration I target the increase between 1980 and 2016, which is 2.11 percentage points (Appendix Figure A8a).[29] For the French calibration I target the increase between 1994 and 2016, which is 2.07 percentage points (Appendix Figure A8b). To calibrate $G(\overline{\phi})$ I target the decline in entry. Entry depends on the share of firms with a higher intangible efficiency because the latter determines what fraction of entrants benefit from the rise of intangibles. For low levels of $G(\bar{\phi})$ there is little chance that an

Figure 3. Number of Products by Firm: Theory and Data



(a) United States



(b) France

Notes: U.S. data are taken from the Compustat Segments file and count the number of primary NAICS codes that firms report to operate in during 1990. Adjusted segments data assign a segment count of 1 for firms that are not included in the segments file. French data are taken from the *Enquête Annuelle de Production dans L'Industrie* (manufacturing only, 2009).

[28] The model is able to analyse the effect of any finite combination of intangible efficiencies. Computational complexity is exponential in the number of different levels of ϕ, however, which is why this calibration sticks to two types.

[29] The increase in all intellectual property investments (including software) except R&D as a percentage of private sector value added was 2.3 percentage points over the same time-frame.

entrant is highly efficient at intangibles. Because high-intangible firms expand strongly, however, entrants are likely to face a high-intangible incumbent when they attempt to enter. This raises effective entry costs and lowers the incentive to enter.[30] In the U.S. calibration, 10.0% of all new firms benefit from a 7.8% higher intangible efficiency. In the French calibration, 6.2% of all new entrants benefit from the high-intangible efficiency, which is 12.9% higher than that of other firms.[31]

I analyse two experiments on the introduction of these high-intangible firms. In the main experiment, I start with an economy where the share of incumbents with $\overline{\phi_j}$ is zero. That is, the rise of high-intangible firms is entirely driven by firms that were not initially operative. This experiment aligns with the observation that the rise of IT-intensity in the 1990s was concentrated in young firms and that the decline of dynamism occurred later for these firms (Haltiwanger et al. 2014). In the alternative experiment, I allow a fraction $G(\overline{\phi})$ of incumbents in the initial balanced growth path to see an improvement in their intangible efficiency from the original, homogeneous, level of efficiency to the higher efficiency $\overline{\phi}$. This experiment assumes that salient differences in intangible efficiency across firms always existed, but that changes in the availability of technology have made these differences relevant. This experiment aligns with the finding that older firms contributed to the speedup and slowdown in productivity growth since the 1990s in Klenow and Li (2020). Besides their difference in narrative, the main difference between both experiments is that the economy transitions faster to the new steady state if a fraction of initial incumbents also has a higher intangible efficiency. The balanced growth paths are identical.

5.2. Results: Balanced Growth Path

The effect of introducing high-intangible efficiency firms is summarized in Table 6. It presents the variables of interest in differences from the original balanced growth path. Two of the changes are targeted: the increase in intangibles as a percentage of value added and the entry rate. The entry rate is well matched, while the share of intangibles in value added is underestimated in the U.S. and overestimated in France. The remainder of Table 6 presents results for untargeted objects. These include the slowdown of productivity growth, the decline in business dynamism and the rise of markups. In the U.S. calibration, the model is able to explain about two-thirds of the rise of markups and half the slowdown of productivity growth. In the French calibration, the model is able to explain all of the decline in the reallocation rate and overshoots the rise of markups. The model predicts a 0.23 percentage-point decline in productivity growth. While this does not explain the

[30] For the U.S., I target the decline in entry in the Business Dynamics Statistics between 1980 and 2016. For France, I impute the decline in entry from the decline in the employment share by entrants in FICUS-FARE, from 1994 to 2016.

[31] While the increase in efficiency of high-intangible firms in France exceeds the efficiency increase at U.S. firms, the level of $\overline{\phi}$ in the U.S. calibration (0.868) still exceeds the level in the French calibration (0.831).

Table 6: Balanced Growth Path Change due to Increase in Intangible Efficiency of Top Firms

	Targeted	United States		France	
		Δ **Model**	Δ **Data**	Δ **Model**	Δ **Data**
Cost Structure					
Intangibles over Value Added	Yes	1.5 pp	2.1 pp	3.6 pp	2.2 pp
Average Fixed-Cost Share	No	3.8 pp	10.6 pp	5.2 pp	4.5 pp
Slowdown of Productivity Growth					
Productivity Growth Rate	No	-0.43 pp	-0.9 pp	-0.23 pp	-1.3 pp
Aggregate R&D over Value Added	No	41.9%	64.5%	67.2%	5.6%
Decline of Business Dynamism					
Entry rate	Yes	-5.8 pp	-5.8 pp	-4.5 pp	-4.5 pp
Reallocation Rate	No	-42.0%	-23%	-23.8%	-23%
Rise of Market Power					
Average Markup	No	21.8 pt	30 pt	22.9 pt	11 pt
Model Wedges					
Labor Wedge	No	8.78 pt	N.A.	11.2 pt	N.A.
Efficiency Wedge	No	0.03 pt	N.A.	0.02 pt	N.A.

Notes: Data columns present the empirical moments, while model columns present the theoretical moments. The change in productivity growth is the difference between growth from 1969-1979 (U.S.) or 1969-1994 (France) to growth post-2005. Other U.S. moments equal the difference between 1980 and 2016. Other French moments equal the difference between 1994 and 2016.

entire lack of growth in France, it does imply a 18% reduction. The model overestimates the decline in the reallocation rate, because all growth in the model occurs through creative destruction.[32]

The bottom of Table 6 presents the change in the labor and efficiency wedge. The labor wedge measures the difference between wages and marginal product, and grew by 8.8 and 11.2 in the U.S. and French calibrations, respectively, due to the rise of markups. The efficiency wedge measures the loss of efficiency from heterogeneity in markups (the final term in (21)), which increases modestly because high-ϕ firms have higher markups than other firms.

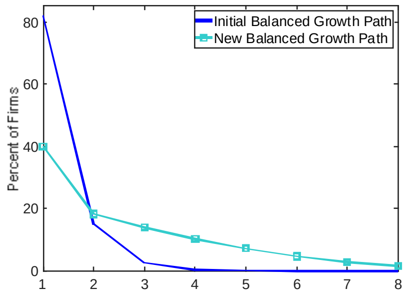
The model predicts a decline in productivity growth despite an *increase* in aggregate research and development, in line with the data in France and the United States.[33] In a model with homogeneous firms this would be paradoxical, because there is a direct relationship between aggregate R&D and growth. Higher investments and lower growth co-exist in this model because innovation activity is concentrated in a smaller group of high-intangible firms, and because some innovations by low-intangible entrants and incumbents fail to enter the market.

The increase in firm concentration is illustrated in Figure 4, which plots the distribution of firms over the number of products that they produce. This is the most direct measure of concentration in the model. The original balanced growth path is characterized by a lower concentration, featur-

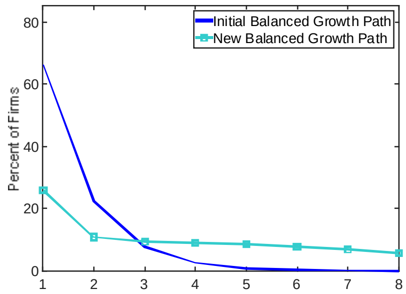
[32]An empirically relevant additional source of innovation is the improvement of goods that firms already produce (e.g. Garcia-Macia et al. 2019, Akcigit and Kerr 2018). In the context of the model, internal innovation would be affected similarly by the rise of intangibles. The rate at which firms innovate depends on the rate at which they discount future profits. This rate is highest for low-intangible firms, which would therefore invest less. High-intangible firms do have a strong incentive to invest in internal innovation. In a model like Peters (2019), however, internal innovation primarily raises a firm's market power, hence furthering the rise of markups and the decline in wages.

[33]The French increase in Table 6 is measured over 1994-2016, while the U.S. increase is over 1980-2016. France experienced a 49.4% increase in R&D over national income between 1980-2016, which is closer to what the model predicts.

Figure 4. Number of Products before and after an Increase in Intangible Efficiency of Top Firms



(a) United States



(b) France

Notes: Lines plot the fraction of firms that produce the number of products on the horizontal axis. Solid lines are from the original calibration. Squared lines present the counterpart for the balanced growth path after the introduction of high-intangible firms.

ing more firms that produce one or two goods than is the case in the new balanced growth path. Conversely, the right tail of the firm-size distribution is fatter, indicating that there are more large firms. Note that the increase in concentration is endogenous: high-intangible firms have higher markups and therefore have more incentives to invest in research and development. This causes them to produce a disproportionate fraction of all goods and to grow larger than other firms.

5.3. Results: Transition Path

The analysis thus far has studied the effect of a rise in intangibles along the balanced growth path. This section shows that short-term dynamics are substantially different. To quantify the transition path, I numerically solve for the path of productivity, markups and wages.[34] This section presents the results from the experiment in which none of the incumbents are assigned a higher intangible efficiency. The alternative experiment, and a comparison with data, is provided in Section 5.4.

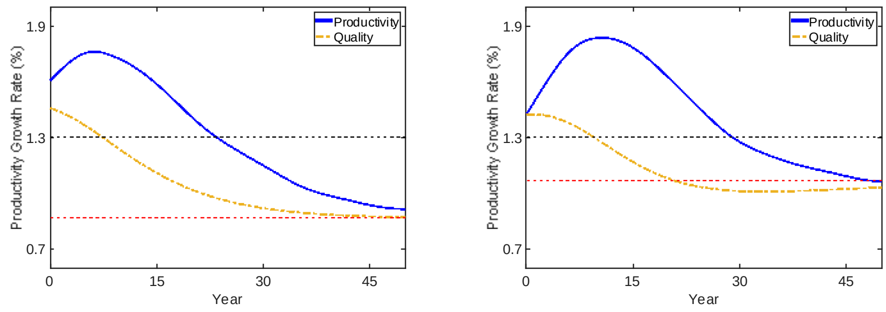
The path of productivity growth is presented in Figure 5. Figure 5a presents results for the U.S. calibration, Figure 5b for the French calibration.[35] The solid blue line plots the path of growth in total factor productivity as defined in (21). The yellow dash-dotted line plots the increase in productivity due to the step-wise improvement of quality, which is the source of long-term growth.

When high-ϕ firms start entering the economy in year 0, there is initially a jump in productivity growth compared to the original steady state (the black upper-dashed line). This is because of a rise in entry, driven by the fact that new firms now have a positive probability of being the profitable high-ϕ type, while the low-ϕ entrants do not face high-ϕ incumbents yet (Figure 6a). As the high-types enter the economy there is a further increase in productivity because they reduce the marginal costs of any good that they produce through the use of intangibles. This causes productivity growth to exceed the growth rate of quality. At peak growth, six years after the introduction of high-ϕ entrants, this boosts growth up to 1.8%. The transitional boom evolves more slowly in

[34]The computational algorithm is described in Appendix E.

[35]Figures in the remainder of this section only plot results for the U.S. calibration because the results are qualitatively similar in both calibrations. Full French results are provided in Appendix F Figure A9.

Figure 5. Transition: Growth Rate of Total Factor Productivity



(a) United States (b) France

Black- and red dashed lines (respectively) indicate the original and the new steady state.

France, because a smaller fraction of start-ups benefit from the higher intangible efficiency (6.2% for France versus 10.0% for the U.S.). The extraordinary growth is predominantly driven by cost reductions from intangibles, consistent with the finding that above-average productivity growth from the mid-1990s to the mid-2000s was primarily caused by IT (Fernald 2015).

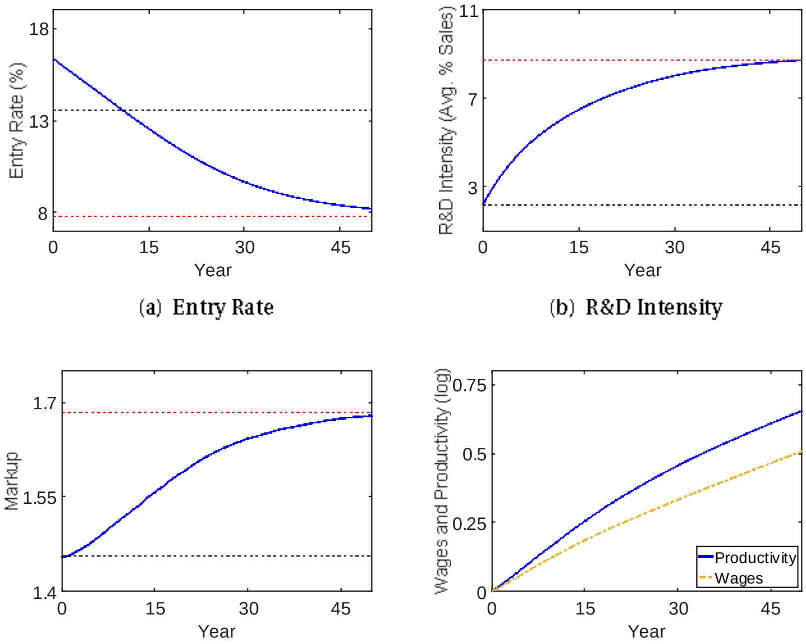
A slowdown occurs from year 7 onwards in the U.S. calibration. Entry declines because high-ϕ incumbents produce an increasingly large share of all products. The probability that an entrant benefits from drawing a high ϕ therefore falls below the probability that it faces a high-ϕ incumbent, which increases the likelihood of a failed innovation.

The decline in productivity growth is mirrored by an *increase* in the average ratio of R&D over sales, also known as R&D intensity (Figure 6b). The increase is large: average R&D intensity increases from 2.5 to 8.8%. This is quantitatively very similar to the data. Among U.S. public firms with positive R&D, the average R&D intensity increased from 2.5 (the calibration target) to 8.7%.[36] It aligns with the result that 'ideas are getting harder to find' in Bloom et al. (2020), who argue that the effect of innovative investments on growth has diminished. The model offers a potential explanation for their result. As high-intangible firms have higher markups, they have a greater incentive to innovate. Because the returns to R&D are concave, these additional investments have limited effects on growth but increase average R&D intensity considerably, causing the decline in research effectiveness. The presence of high-ϕ incumbents further means that a fraction of the innovations fail to be introduced to the market, again diminishing the effect of research on growth.

The model also sheds light on why wages did not keep up with productivity growth in the past 20 years, which has caused a decline in the labor share (Kehrig and Vincent 2020). While the reallocation of economic activity to higher-ϕ firms leads to a reduction of marginal costs and an increase in productivity, there is no increase in wages because productivity is offset by higher markups (Figure 6c). Note that markups increase because activity reallocates towards high-markup firms, in line with empirical evidence (e.g. Baqaee and Farhi 2020, Autor et al. 2020). This leads to a decoupling

[36] R&D intensity among all public firms increased from 2.0 to 6.7%, again similar to the increase in the model. French R&D expenditure over sales increased from 3.1% among positive spenders (the calibration target) to 4.0%.

Figure 6. Transition Path for Entry, R&D, Markups, Wages



(a) Entry Rate (b) R&D Intensity

(c) Markups (d) Wages and Productivity

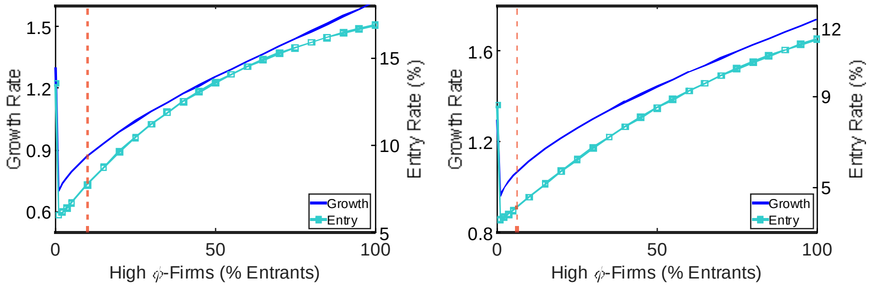
Black- and red dashed lines (respectively) in (a) to (c) indicate the original and the new steady state. Figure (a) presents the entry rate, (b) the average ratio of R&D to sales, (c) the average markup, (d) the path of wages (dashed yellow) and productivity (solid blue).

of wages and productivity (Figure 6d). Wages continue to grow at the rate of quality improvements, but do not benefit from the transitory increase in productivity growth from intangible adoption.

The welfare effect of the rise of intangibles is given by the change in the discounted sum of log consumption. Two counteracting effects are at play. The initial boom in growth raises the level of productivity, which is positive for welfare. The subsequent slowdown of productivity growth lowers output, which reduces welfare. The permanent rise of R&D worsens this negative effect, because a smaller fraction of the labor force is dedicated to the production of consumption goods.

The model predicts that utility falls by 2.3% in the U.S. calibration and by 1% in the French calibration. The decline is modest because consumers place greater weight on current consumption, which is boosted by the initial spike in productivity growth. The welfare effect is determined by the fraction of firms that have access to the higher intangible efficiency, $G(\overline{\phi})$. The effect of $G(\overline{\phi})$ on growth and entry is illustrated in Figure 7. At $G(\overline{\phi}) = 0$, the economy is in the original steady state. As the share of entrants with a high-intangible efficiency becomes positive there is a substantial decline in growth and entry. This is because the smaller $G(\overline{\phi}) > 0$, the greater the increase in variance and the smaller the increase of the expected intangible efficiency. If all firms see an increase in ϕ, then average markups would increase as would the incentive to innovate. A sufficiently homogeneous increase in intangible efficiency therefore raises entry and growth above the

Figure 7. Balanced Growth Path Effects of an Increase in Intangible Efficiency for Top Firms



(a) **United States** (b) **France**

Notes: Balanced growth path growth- and entry rates for various levels of $G(\overline{\phi})$. Figure 7a plots the U.S. calibration, in which $\overline{\phi}$ exceeds the ϕ of other firms by 7.8%. Figure 7b plots results for the French calibration, in which $\overline{\phi}$ exceeds the ϕ of other firms by 12.9%. Red-dashed lines present $G(\overline{\phi})$ in the calibration of Table 6. The lowest $G(\overline{\phi}) > 0$ plotted is 1%.

old steady-state level.[37] Conversely, a mean-preserving spread of ϕ has a negative effect on growth because it reduces incentives to enter.

Welfare changes under alternative calibrations for $G(\overline{\phi})$ are summarized in Table 7. In the main exercise, 6% (10%) of U.S. (French) firms receive the high efficiency. If that fraction is increased to 50%, the U.S. and French calibrations respectively display an *increase* in welfare by 1.4% and 3.6%.

5.4. Alternative Experiment: Incumbents Receive High Intangible Efficiency

The previous section plotted the transition path when all incumbents retain their original intangible efficiency, such that only new firms are assigned $\overline{\phi}$. I now analyse the transition path when, on top of a fraction $G(\overline{\phi})$ of entrants, the same fraction of initial incumbents receives the higher intangible efficiency. This experiment represents, for example, the case in which heterogeneous intangible efficiency is a salient feature of firms, which only becomes relevant when technological advancement enables the use of intangible inputs to reduce marginal costs.

Results for the U.S. calibration are presented in the left-hand plots of Figure 8, which present results from the previous section (solid blue lines) and the alternative experiment (dashed yellow lines). Figure 8a plots productivity growth. When a fraction of incumbents receives a high-intangible efficiency, the initial increase in growth is larger. These firms immediately become more profitable because their markups jump up (Figure 8c), which drives firms to raise their R&D expen-

Table 7: Welfare Change at Various Levels of Intangible Adoption

	United States					France				
$G(\overline{\phi})$:	0.06	0.10	0.25	0.50	1.0	0.06	0.10	0.25	0.50	1.0
Δ Welfare	-2.52%	-2.28%	-0.76%	1.41%	3.85%	-1.00%	-0.47%	1.31%	3.57%	6.58%

Notes: Percent change from original balanced growth path. $G(\overline{\phi}) = 0.10$ for the U.S. and $G(\overline{\phi}) = 0.062$ for France in the main analysis.

[37] In Figure 7 this happens when around 45% of entrants receive the higher efficiency in both calibrations. Note that this is an exaggeration because the figure does not correct for the fact that the increase in the model's steady-state intangible share would exceed the empirical increase when a larger fraction of entrants receive $\overline{\phi}$.

ditures. This is visible in the path of average incumbent-R&D over sales (Figure 8e). Growth converges to its steady state level more swiftly because the distribution of firm types at year 0 is closer to the distribution along the new balanced growth path. The increase in entry is muted because high-intangible incumbents immediately use their cost advantage to undercut low-ϕ entrants. Overall, the model's predictions are similar with the inclusion of high-intangible incumbents. The relative timing of the trends in productivity, business dynamism and market power is largely unchanged.

The figures on the right-hand side of Figure 8 plot empirical counterparts to the transition path. The horizontal axes start in 1985 and span 45 years, to match the theoretical plots. The model is largely able to explain the quantitative features of the data. The model predicts that it takes approximately 45 years for entry rates and markups to converge to the new steady state, while convergence in the data takes 30 years. By that time, however, the theoretical series have approached levels that are close to their new steady states. R&D intensity increases faster in the data.

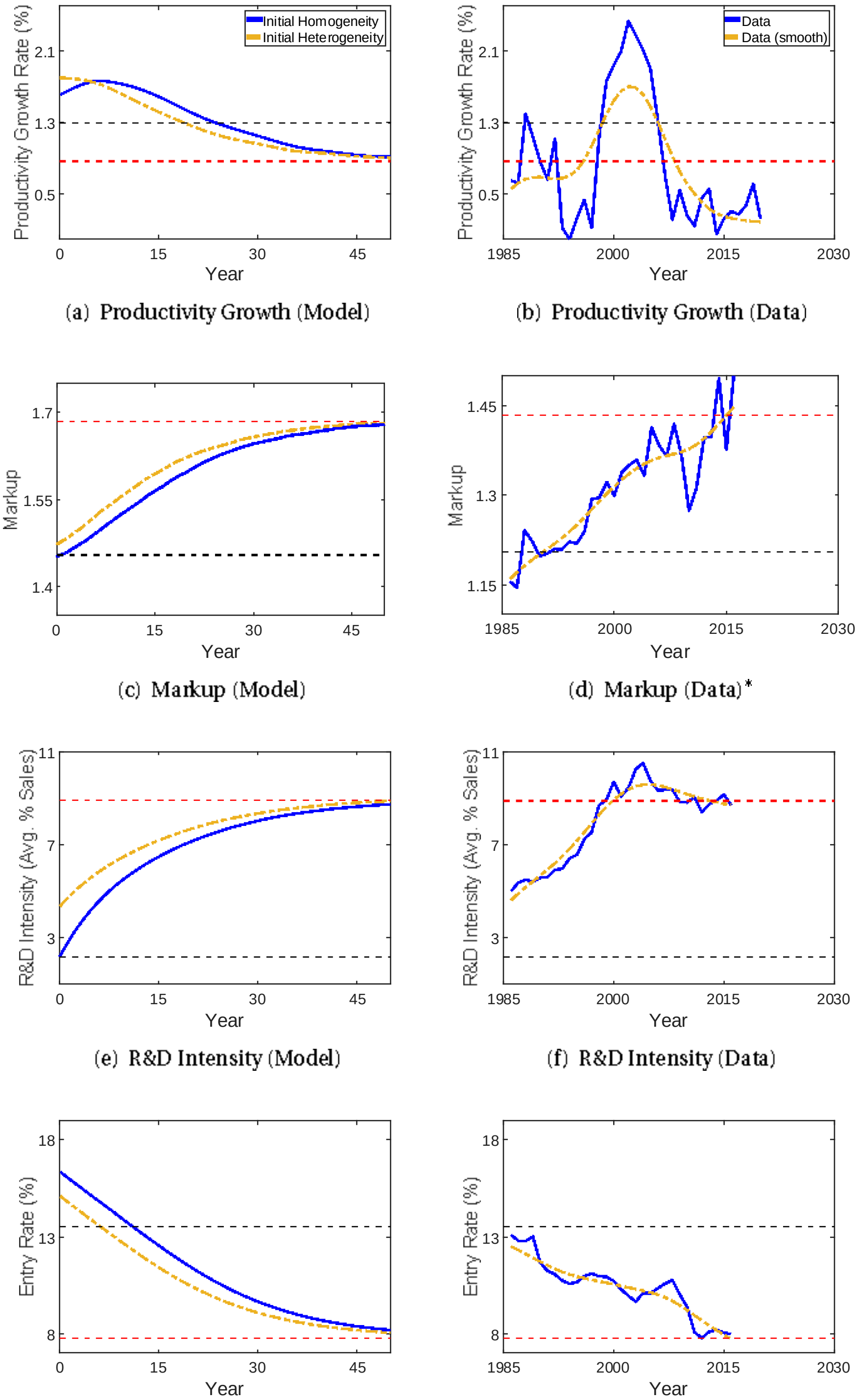
Productivity growth in the data contains the initial boom and subsequent decline, although its timing differs from the model. The boom occurs between 1995 and 2002, while the model predicts a boom right after high-intangible firms are introduced. Note, however, that no part of the transition path is targeted. The magnitude of the boom, with growth spiking at 1.8% in the model and 1.7% in the smoothed data, is similar. The boom also lasts for six years in both the model and the data. The model is therefore capable of replicating most quantitative features of the path of productivity growth. Its predictions for business dynamism and market power are closer to the data when a fraction of incumbents also receive a higher intangible efficiency (yellow dashed lines), although the path of productivity is closer to the data when it is only awarded to entrants (blue solid lines).

Figure A10 in Appendix F presents the French transition path, which is qualitatively similar. An empirical comparison is complicated by the fact that data on entry and business dynamism are only available from 1994, such that the effects of intangibles are likely to predate the figures. The ability of the model to fit the time path of productivity growth is worse than for the U.S., furthermore, because productivity growth in France was negative for most years after 2005. Between 1994 and 2016 the model performs well at replicating the rate of decline in entry and of the rise of markups, although R&D expenditures rise significantly faster in the model than in the data.

6. Extensions

This section explores two extensions. I first show that the model's predictions for productivity growth and business dynamism also hold if markups are constant. I then show that the results in the previous section are robust when firms internalize the diminishing option value of innovation.

Figure 8. Transition Path: Model Predictions versus Data



(a) Productivity Growth (Model)

(b) Productivity Growth (Data)

(c) Markup (Model)

(d) Markup (Data)*

(e) R&D Intensity (Model)

(f) R&D Intensity (Data)

(g) Entry Rate (Model)

(h) Entry Rate (Data)

Black- and red dashed lines (respectively) indicate the original and the new steady state. U.S. calibration. Productivity growth in Figure 8b is a 5-year centred moving average to reduce noise. HP-filter smoothing parameter is 100. Data sources: productivity growth from Fernald (FRBSF), R&D from Compustat, entry from the BDS, markups from Compustat.

* The axis for the markup data is re-scaled by subtracting 0.25 from the model's original and final steady state. This is because the initial level of the markup is untargeted, and the model's markup is 0.25 lower than the empirical markup in the original steady state.

Table 8: Comparison of Steady States - Constant Markup

	United States			France		
	Δ Model (Var. μ)	**Δ Model** (Fixed $\bar{\mu}$)	**Δ Data**	**Δ Model** (Var. μ)	**Δ Model** (Fixed $\bar{\mu}$)	**Δ Data**
Cost Structure						
Average Fixed-Cost Share	3.8 pp	4.2 pp	10.6 pp	5.2 pp	5.7 pp	4.5 pp
Intangibles over Value Added	1.5 pp	4.7 pp	2.1 pp	3.6 pp	6.5 pp	2.2 pp
Slowdown of Productivity Growth						
Productivity Growth Rate	-0.43 pp	-0.67 pp	-0.9 pp	-0.23 pp	-0.74 pp	-1.3 pp
Aggregate R&D over Value Added	41.9%	46.8%	64.5%	67.2%	-62.8%	5.6%
Decline of Business Dynamism						
Entry rate	-5.8 pp	-6.7 pp	-5.8 pp	-4.5 pp	-3.3 pp	-4.5 pp
Reallocation Rate	-42.0%	-59.8%	-23%	-23.8%	-64.5%	-23%

Notes: Data columns present the empirical moments, while model columns present the theoretical moments. Model - $\bar{\mu}$ columns present theoretical moments where markups are exogenous and homogeneous across firms in both steady states. The change in productivity growth is the difference between growth from 1969-1994 (France) or 1969-1979 (U.S.) to growth post-2005. Other French moments equal the difference between values in 1994 and in 2016. Other U.S. moments equal the difference between 1980 and 2016.

6.1. Constant Markups

The analysis thus far has explained the decline in productivity growth and business dynamism jointly with the rise of markups. Recent evidence shows that the labor share in Europe is constant outside of the residential housing sector (Gutierrez and Piton 2020), while markups may be hard to measure accurately in the absence of data on prices (e.g. Bond et al. 2020).[38]

This section shows that the model predicts a *larger* decline in productivity growth if markups are constant. To do so, I impose that all firms charge a constant markup $\bar{\mu}$ over their marginal costs. The markup is calibrated to match the average endogenous markup of 1.22 in the French- and 1.47 in the U.S. calibration of the model. The remainder of the model is left unchanged. In particular, I do not alter the demand system to endogenously arrive at a fixed markup. This facilitates a direct comparison with the main results.[39] The first-order condition for intangibles reads

$$s_{ij} = 1 - \left(w \cdot Y^{-1} \cdot \psi \cdot (1-\phi_i) \cdot \bar{\mu}\right)^{\frac{1}{\psi}},$$

which follows from inserting the new pricing rule into first-order condition (13). Because markups are homogeneous, the expressions for output and wages respectively simplify to

$$Y = \exp\left(\int_0^1 \int \mathbf{1}_{j\in \mathcal{J}_i} \ln\left[\frac{q_{ij}}{1-s_{ij}}\right] \, di dj\right) \cdot L^p, \text{ and } w = \exp\left(\int_0^1 \int \mathbf{1}_{j\in \mathcal{J}_i} \ln\left[\frac{q_{ij}}{1-s_{ij}}\right] \, di dj\right) \cdot \bar{\mu}^{-1}.$$

Table 8 compares the change in the steady-state values in the model with variable markups (columns headed Var. μ) and constant markups (columns headed Fixed $\bar{\mu}$). The rise of high-intangible firms causes productivity growth to fall significantly more when markups are constant:

[38] Appendix C discusses the model's robustness to measurement issues in markups in the absence of price data.

[39] The introduction of CES utility, for example, would require functional-form changes in order to maintain a balanced growth path. In particular, the fixed-cost function (6) would have to be multiplied by a product's relative quality.

growth now falls by 0.7 percentage points in both calibrations. When markups are endogenous, high-intangible firms are profitable and invest strongly in R&D. This offsets a part of the decline in growth induced by the fact that high-intangible firms undercut other firms on price. When markups are exogenous there is no motive for R&D by high-intangible firms, worsening the decline in growth. Reallocation rates mirror the additional decline in productivity growth when markups are constant, and now fall well in excess of their empirical decline. The rise of intangible expenditures over value added is substantially larger, as wages are higher under constant markups.

Table 8 is the first table to display a qualitative difference between the U.S. and the French calibration. While the French calibration now predicts an intuitive decline in aggregate R&D, the U.S. calibration still shows a 46.8% increase. This is driven by R&D spending of *low-intangible* firms. Because constant markups limit the increase in R&D by high-intangible firms, low-intangible incumbents are less likely to be challenged. Combined with the decline in entry, this reduces the rate at which low-intangible firms discount successful innovations, raising the present value of innovation. In the French calibration, this effect is not sufficiently large to prevent a decline in aggregate R&D. In the U.S. calibration it is sufficiently large, explaining the divergence.

6.2. Value Function Specification

The preceding analysis relied on a simplified dynamic optimization problem where firms did not internalize the change in their innovation capacity when they added a new product to their portfolio. This assumption significantly improves tractability, as it allows for a closed-form expression of the first-order conditions for innovation. This section shows that the results are qualitatively and quantitatively robust to removing this assumption. The new value function is characterized by

$$rV_t(\phi_i,\tilde{J}_i) - \dot{V}_t(\phi_i,\tilde{J}_i) = \max_{x_i}\left\{\begin{array}{c} \sum_{j\in\tilde{J}_i}\pi_t(\phi_i,\lambda_{ij}) + \tau(\phi_i)\cdot\left[V_t(\phi_i,\tilde{J}_i\setminus\{\lambda_{ij}\}) - V_t(\phi_i,\tilde{J}_i)\right] \\ +x_i\cdot\text{Prob}\left(\lambda_{ij}\geq\frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})}-1\right) \\ \cdot\mathbb{E}_{\phi_i}\left[V_t(\phi_i,\tilde{J}_i\cup_+\lambda_{ij}) - V_t(\phi_i,\tilde{J}_i)\right] - w_t\eta_x(x_i)^{\psi_x}n_i^{-\sigma}). \end{array}\right\}.$$

The solution of this function is considerably less tractable than the solution in Section 3 because the function no longer scales linearly in firm size. As firms get larger, the option value of investing in R&D increases, causing them to choose a higher innovation rate. R&D does not fully scale with size, however, because the parameter σ is estimated such that the model matches the negative empirical relationship between firm size and growth. Proposition 3 summarizes the new solution:

Proposition 3. *The value function of a firm with intangible efficiency ϕ_i that produces a portfolio of goods $\tilde{J}_i$ with cardinality n_i grows at rate g along the balanced growth path and is given by*

$$V_t(\phi_i,\tilde{J}_i) = \sum_{j\in\tilde{J}_i}\Upsilon_t^1(\phi_i,\lambda_{ij}) + \Upsilon_{t,n_i}^2(\phi_i),$$

Table 9: Comparison of Steady States - Alternative Value Function Specification

	United States			France		
	Δ Model (Main)	**Δ Model** (Full Val.)	**Δ Data**	**Δ Model** (Main)	**Δ Model** (Full Val.)	**Δ Data**
Cost Structure						
Average Fixed-Cost Share	3.8 pp	3.7 pp	10.6 pp	5.2 pp	5.1 pp	4.5 pp
Intangibles over Value Added	1.5 pp	1.4 pp	2.1 pp	3.6 pp	3.7 pp	2.2 pp
Slowdown of Productivity Growth						
Productivity Growth Rate	-0.43 pp	-0.41 pp	-0.9 pp	-0.23 pp	-0.18 pp	-1.3 pp
Aggregate R&D over Value Added	41.9%	49.4%	64.5%	67.2%	73.5%	5.6%
Decline of Business Dynamism						
Entry rate	-5.8 pp	-5.0 pp	-5.8 pp	-4.5 pp	-4.9 pp	-4.5 pp
Reallocation Rate	-42.0%	-39.4%	-23%	-23.8%	-18.7%	-23%
Rise of Market Power						
Average Markup	21.8 pt	20.9 pt	30 pt	22.9 pt	22.0 pt	11 pt
Model Objects						
Labor Wedge	8.8 pt	8.5 pt	N.A.	11.2 pt	10.9 pt	N.A.
Efficiency Wedge	.03 pt	.04 pt	N.A.	.02 pt	0.02 pt	N.A.

Notes: Data columns present the empirical moments, while Model - Main columns present the theoretical moments from the model in the main analysis. Model - Full Val. columns present moments where the value function includes the R&D option value. The change in productivity growth is the difference between growth from 1969-1994 (France) or 1969-1979 (U.S.) to growth post 2005. Other French moments equal the difference between values in 1994 and in 2016. Other U.S. moments equal the difference between 1980 and 2016.

where Υ_1 is the present value of the profit flow from producing good j. Matching coefficients gives

$$\Upsilon^1(\phi_i,\lambda_{ij}) = \pi_t(\phi_i,\lambda_{ij})\cdot(r-g+\tau(\phi))^{-1},$$

while Υ_{2n_i} is the option value of research and development which evolves along this sequence:

$$\begin{aligned}\Upsilon^2_{t,n_i+1}(\phi_i) \quad = \quad & \Big[\Big((r-g)\cdot\Upsilon^2_{t,n_i}(\phi_i)+n_i\cdot\tau(\phi_i)\cdot\big[\Upsilon^2_{t,n_i}(\phi_i)-\Upsilon^2_{t,n_i-1}(\phi_i)\big]\psi^x-1\Big)\cdot(\psi^x-1)^{-1}\Big]^{\frac{\psi^x-1}{\psi^x}} \\ & \cdot\text{Prob}\left(\lambda_{ij}\geq\frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})}-1\right)^{-1}\cdot\psi^x\cdot(\eta\cdot w_t)^{\psi^{x-1}}\cdot n_i^{-\frac{\sigma}{\psi^x}}+\Upsilon^2_{t,n_i}(\phi_i)-\Upsilon^1_t(\phi_i,\lambda_{ij}),\end{aligned}$$

such that the first-order conditions for optimal research and development and entry read

$$x(\phi_i,n_i)=\left(\text{Prob}\left(\lambda_{ij}\geq\frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})}-1\right)\cdot\frac{\mathbb{E}_{\phi_i}\left[\Upsilon^1_t(\phi_i,\lambda_{ij})+\Upsilon^2_{t,n_i+1}(\phi_i)-\Upsilon^2_{t,n_i}(\phi_i)\right]}{\eta^x\cdot\psi^x\cdot w_t}\right)^{\frac{1}{\psi^x-1}}\cdot n_i^{\frac{\sigma}{\psi^x-1}},$$

$$e=\left(\sum_{\phi_h\in\Phi}G(\phi_h)\cdot\text{Prob}\left(\lambda_{ij}\geq\frac{p^{choke}(\phi_h)}{p^{choke}(\phi_{-i})}-1\right)\cdot\frac{\mathbb{E}_{\phi_h}\left[\Upsilon^1_t(\phi_i,\lambda_{ij})+\Upsilon^2_{t,1}(\phi_i)\right]}{\eta^e\cdot\psi^e\cdot w_t}\right)^{\frac{1}{\psi^e-1}}. \tag{25}$$

Proof: Appendix A.

I perform the same experiment as in Section 5.1. To ease the comparison with the main analysis, I retain most of the previous calibration. I re-estimate σ such that the model matches the empirical relationship between firm size and growth. Under an unchanged calibration, the model would predict a strongly negative relationship between firm-growth and firm-size. This is because firms now internalize that the additional option value from producing a good diminishes in n_i. Appendix Table A5 details the new model's calibration and main moments. Compared to the original calibration, there is an increase in the value of σ for both France and the U.S. The higher parameter value ensures that the empirical deviation from Gibrat's Law is still matched by the model.

Table 9 compares the effect of introducing a group of high-intangible firms in the model with the new value function specification to the effect in the main analysis. Both specifications of the model predict a decline in productivity growth by 0.4 percentage points in the U.S. and 0.2 percentage points in France. The predicted declines in entry are also similar, as are the changes in the reallocation rate. The increase in average markups is slightly smaller in the United States in the new specification because high-intangible firms occupy a slightly smaller fraction of all products in equilibrium. Conditional on the recalibration of σ, the model displays a similar relationship between firm-size and firm-growth. Because the value function specification in this section differs from the value function in the main analysis only in this regard, the results are both qualitatively and quantitatively robust to the use of the full value function.

7. Conclusion

This paper proposes a unified explanation for the decline of productivity growth, the fall in business dynamism and the rise of markups. I hypothesize that the rise of intangible inputs — in particular, information technology and software — can explain these trends. Central to the theory is that intangible inputs shift costs from variable to fixed costs, and that firms differ in the efficiency with which they deploy these inputs.

I embed intangibles in an endogenous growth model with heterogeneous multi-product firms, variable markups and realistic entry and exit dynamics. The model suggests that when a subset of new firms becomes more efficient at using intangible inputs, the aggregate rise of intangibles is accompanied by a decline in both entry and long-term growth. I structurally estimate the model to match administrative micro data on U.S. listed firms and the universe of French firms, and find that intangibles cause a decline of long-term productivity growth of 0.4 percentage points in the U.S. calibration and 0.2 percentage points in the French calibration. Despite the decline of growth, there is an increase in R&D expenditures, in line with empirical evidence. Research and development becomes less effective because it is concentrated among a small number of firms and because a fraction of innovators are unable to beat high-intangible incumbents.

While the rise of intangibles negatively affects growth in the long run, its short-run effect is positive. By numerically solving the transition path between the original and the new balanced growth path, I show that growth initially increases for six years. This is because firms with high-intangible efficiencies initially disrupt sectors by producing goods at lower marginal costs. The overall effect on consumption is negative, although technologies that raise the diffusion of intangible inputs across firms yield significant welfare gains.

References

Acemoglu, D., Akcigit, U., Alp, H., Bloom, N., and Kerr, W. (2018). Innovation, reallocation, and growth. *American Economic Review*, 108(11):3450–91.

Adler, G., Duval, M. R. A., Furceri, D., Sinem, K., Koloskova, K., Poplawski-Ribeiro, M., et al. (2017). *Gone with the Headwinds: Global Productivity*. International Monetary Fund.

Aghion, P., Bergeaud, A., Boppart, T., Klenow, P. J., and Li, H. (2019). A theory of falling growth and rising rents. *Federal Reserve Bank of San Francisco Working Paper, November 2019*.

Aghion, P. and Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, 60(2):323–351.

Akcigit, U. and Kerr, W. R. (2018). Growth through heterogeneous innovations. *Journal of Political Economy*, 126(4):1374–1443.

Akicigit, U. and Ates, S. (2019). Ten facts on declining business dynamism and lessons from endogenous growth theory. *NBER working paper*, 25755.

Amiti, M., Itskhoki, O., and Konings, J. (2019). International Shocks, Variable Markups, and Domestic Prices. *The Review of Economic Studies*.

Andrews, D., Criscuolo, C., Gal, P. N., et al. (2016). The best versus the rest: the global productivity slowdown, divergence across firms and the role of public policy. Technical report, OECD.

Appelt, S., Bajgar, M., Criscuolo, C., and Galindo-Rueda, F. (2020). The heterogeneous effects of r&d tax incentives and the role of policy mix. *Forthcoming OECD Working Paper*.

Aquilante, T., Chowla, S., Dacic, N., Haldane, A., Masolo, R., Schneider, P., Seneca, M., and Tatomir, S. (2019). Market power and monetary policy. *Bank of England Working Paper*.

Atkeson, A. and Burstein, A. (2019). Aggregate implications of innovation policy. *Journal of Political Economy*, 127(6):2625–2683.

Autor, D., Dorn, D., Katz, L. F., Patterson, C., and Van Reenen, J. (2020). The fall of the labor share and the rise of superstar firms. *The Quarterly Journal of Economics*, 135(2):645–709.

Ayyagari, M., Demirguc-Kunt, A., and Maksimovic, V. (2018). Who are america's star firms? *Policy Research working paper no. WPS 8534, World Bank Group*.

Bajgar, M., Criscuolo, C., and Timmis, J. (2019). Supersize me: Intangibles and industry concentration. *mimeo*.

Baqaee, D. R. and Farhi, E. (2020). Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics*, 135(1):105–163.

Barkai, S. (2020). Declining labor and capital shares. *Journal of Finance, Forthcoming*.

Bartel, A., Ichniowski, C., and Shaw, K. (2007). How does information technology affect productivity? plant-level comparisons of product innovation, process improvement, and worker skills. *The quarterly journal of Economics*, 122(4):1721–1758.

Basu, S. (2019). Are price-cost markups rising in the united states? a discussion of the evidence. *Journal of Economic Perspectives*, 33(3):3–22.

Bessen, J. E. (2017). Information technology and industry concentration. *Boston Univ. School of Law, Law and Economics Research Paper No. 17-41, April 2018*.

Bessen, J. E. and Righi, C. (2019). Shocking technology: What happens when firms make large it investments? *Boston Univ. School of Law, Law and Economics Research Paper*, (19-6).

Bijnens, G. and Konings, J. (2018). Declining business dynamism in belgium. *Small Business Economics*, pages 1–39.

Bloom, N., Brynjolfsson, E., Foster, L., Jarmin, R., Patnaik, M., Saporta Eksten, I., and Van Reenen, J. (2014). It and management in america. *Working Paper, February 2014*.

Bloom, N., Griffith, R., and Van Reenen, J. (2002). Do r&d tax credits work? evidence from a panel of countries 1979–1997. *Journal of Public Economics*, 85(1):1–31.

Bloom, N., Jones, C. I., Van Reenen, J., and Webb, M. (2020). Are ideas getting harder to find? *American Economic Review*, 110(4):1104–44.

Bloom, N., Sadun, R., and Van Reenen, J. (2012). Americans do it better: Us multinationals and the productivity miracle. *American Economic Review*, 102(1):167–201.

Bond, S., Hashemi, A., Kaplan, G., and Zoch, P. (2020). Some unpleasant markup arithmetic: Production function elasticities and their estimation from production data. Technical report, NBER.

Bresnahan, T. F., Brynjolfsson, E., and Hitt, L. M. (2002). Information technology, workplace organization, and the demand for skilled labor: Firm-level evidence. *The Quarterly Journal of Economics*, 117(1):339–376.

Brynjolfsson, E., Rock, D., and Syverson, C. (2020). The productivity j-curve: How intangibles complement general purpose technologies. *American Economic Journal: Macro, Forthcoming*.

Burstein, A., Carvalho, V., and Grassi, B. (2019). Bottom-up markup fluctuations. *Working Paper, October 2018*.

Caballero, R. J., Farhi, E., and Gourinchas, P.-O. (2017). Rents, technical change, and risk premia accounting for secular trends in interest rates, returns on capital, earning yields, and factor shares. *American Economic Review*, 107(5):614–20.

Calligaris, S., Criscuolo, C., Marcolin, L., et al. (2018). Mark-ups in the digital era. Technical report, OECD Publishing.

Calvino, F., Criscuolo, C., and Menon, C. (2016). No country for young firms? *OECD Working Paper, April 2016*.

Cavalleri, M. C., Eliet, A., McAdam, P., Petroulakis, F., Soares, A. C., and Vansteenkiste, I. (2019). Concentration, market power and dynamism in the euro area. *ECB Working Paper*.

Cavenaile, L., Celik, M. A., and Tian, X. (2020). Are markups too high? competition, strategic innovation, and industry dynamics. *Working Paper*.

Corrado, C., Hulten, C., and Sichel, D. (2009). Intangible capital and us economic growth. *Review of income and wealth*, 55(3):661–685.

Crespi, G., Criscuolo, C., and Haskel, J. (2007). Information technology, organisational change and productivity growth: Evidence from uk firms. *CEP Discussion Paper No 783, March 2007*.

Crouzet, N. and Eberly, J. (2018). Understanding weak capital investment: The role of market concentration and intangibles. *prepared for the Jackson Hole Economic Policy Symposium*.

Davis, S. J., Haltiwanger, J., Jarmin, R., Miranda, J., Foote, C., and Nagypal, E. (2006). Volatility and dispersion in business growth rates: Publicly traded versus privately held firms [with comments and discussion]. *NBER macroeconomics annual*, 21:107–179.

De Loecker, J., Eeckhout, J., and Unger, G. (2020). The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics*, 135(2):561–644.

De Loecker, J. and Warzynski, F. (2012). Markups and firm-level export status. *The American Economic Review*, 102(6):2437–2471.

Decker, R., Haltiwanger, J., Jarmin, R., and Miranda, J. (2014). The role of entrepreneurship in us job creation and economic dynamism. *Journal of Economic Perspectives*, 28(3):3–24.

Decker, R. A., Haltiwanger, J., Jarmin, R. S., and Miranda, J. (2016). Where has all the skewness gone? the decline in high-growth (young) firms in the us. *European Economic Review*, 86:4–23.

Decker, R. A., Haltiwanger, J., Jarmin, R. S., and Miranda, J. (2017). Declining dynamism, allocative efficiency, and the productivity slowdown. *American Economic Review*, 107(5):322–26.

Decker, R. A., Haltiwanger, J. C., Jarmin, R. S., and Miranda, J. (2018). Changing business dynamism and productivity: Shocks vs. responsiveness. *NBER Working Paper, January 2018*.

Diez, F., Fan, J., and Villegas-Sanchez, C. (2019). Global declining competition. *IMF Working Paper, April 2019*.

Döttling, R., Gutiérrez Gallardo, G., and Philippon, T. (2017). Is there an investment gap in advanced economies? if so, why? *ECB Forum on Central Banking Conference Proceedings*.

Edmond, C., Midrigan, V., and Xu, D. Y. (2019). How costly are markups? *Working Paper, January 2019*.

Farhi, E. and Gourio, F. (2018). Accounting for macro-finance trends: Market power, intangibles, and risk premia. *Brookings Papers on Economic Activity, Fall 2018*.

Fernald, J. G. (2015). Productivity and potential output before, during, and after the great recession. *NBER macroeconomics annual*, 29(1):1–51.

García-Feijóo, L. and Jorgensen, R. D. (2010). Can operating leverage be the cause of the value premium? *Financial Management*, 39(3):1127–1154.

Garcia-Macia, D., Hsieh, C.-T., and Klenow, P. J. (2019). How destructive is innovation? *Econometrica*, 87(5):1507–1541.

Gutierrez, G. and Philippon, T. (2017). Investment-less growth: An empirical investigation. *Brookings Papers on Economic Activity*, pages 89–190.

Gutiérrez, G. and Philippon, T. (2019). The failure of free entry. Technical report, NBER.

Gutierrez, G. and Piton, S. (2020). Revisiting the global decline of the (non-housing) labor share. *American Economic Review: Insights, Forthcoming*.

Hall, B. H., Mairesse, J., and Mohnen, P. (2010). Measuring the returns to r&d. *Handbook of the Economics of Innovation*, 2:1033–1082.

Hall, R. E. (1988). The relation between price and marginal cost in us industry. *Journal of political Economy*, 96(5):921–947.

Haller, S. A. and Siedschlag, I. (2011). Determinants of ict adoption: Evidence from firm-level data. *Applied Economics*, 43(26):3775–3788.

Haltiwanger, J., Hathaway, I., and Miranda, J. (2014). Declining business dynamism in the us high-technology sector.

Haskel, J. and Westlake, S. (2017). *Capitalism without capital: the rise of the intangible economy*. Princeton University Press.

Hopenhayn, H., Neira, J., and Singhania, R. (2018). The rise and fall of labor force growth: Implications for firm demographics and aggregate trends. *Working Paper*.

Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica*, pages 1127–1150.

Hsieh, C.-T. and Rossi-Hansberg, E. (2019). The industrial revolution in services. *Working Paper*.

IMF (2019). The rise of corporate market power and its macroeconomic effects. *World Economic Outlook Chapter 2*.

Karabarbounis, L. and Neiman, B. (2019). Accounting for factorless income. *NBER Macroeconomics Annual*, 33(1):167–228.

Kehrig, M. and Vincent, N. (2019). Good dispersion, bad dispersion.

Kehrig, M. and Vincent, N. (2020). Growing productivity without growing wages: The micro-level anatomy of the aggregate labor share decline.

Klenow, P. J. and Li, H. (2020). Innovative growth accounting. In *NBER Macroeconomics Annual 2020, volume 35*. University of Chicago Press.

Klette, T. J. and Kortum, S. (2004). Innovating firms and aggregate innovation. *Journal of political economy*, 112(5):986–1018.

Korinek, A. and Ng, D. X. (2019). Digitization and the macroeconomics of superstars. *Working Paper*.

Lashkari, D., Bauer, A., and Boussard, J. (2019). Information technology and returns to scale. *Working Paper*.

Lentz, R. and Mortensen, D. T. (2008). An empirical model of growth through product innovation. *Econometrica*, 76(6):1317–1373.

Lev, B. (1974). On the association between operating leverage and risk. *Journal of financial and quantitative analysis*, 9(4):627–641.

Li, W. C. and Hall, B. H. (2020). Depreciation of business r&d capital. *Review of Income and Wealth*, 66(1):161–180.

Liu, E., Mian, A., and Sufi, A. (2019). Low interest rates, market power, and productivity growth. Technical report.

Martinez, J. (2019). Automation, growth and factor shares. *Working Paper, March 2019*.

McKinsey (2018). Superstars': The dynamics of firms, sectors, and cities leading the global economy. *McKinsey Global Institute (MGI) Discussion Paper*.

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725.

Mian, A., Rao, K., and Sufi, A. (2013). Household Balance Sheets, Consumption, and the Economic Slump. *The Quarterly Journal of Economics*, 128(4):1687–1726.

Peters, M. (2019). Heterogeneous mark-ups, growth and endogenous misallocation. *Econometrica, Forthcoming*.

Peters, M. and Walsh, C. (2019). Declining dynamism, increasing markups and missing growth: The role of the labor force. *Working Paper*.

Pugsley, B. W. and Şahin, A. (2018). Grown-up business cycles. *The Review of Financial Studies*, 32(3):1102–1147.

Saibene, G. (2017). Operating leverage and the decline of business dynamism. *Mimeo*.

Schivardi, F. and Schmitz, T. (2019). The it revolution and southern europe's two lost decades. *Journal of the European Economic Association*.

Syverson, C. (2019). Macroeconomics and market power: Context, implications, and open questions. *Journal of Economic Perspectives*, 33(3):23–43.

Traina, J. (2018). Is aggregate market power increasing? production trends using financial statements. *mimeo*.

Van Reenen, J. (2018). Increasing differences between firms: market power and the macro-economy. *prepared for the Jackson Hole Economic Policy Symposium*.

Veugelers, R., Rückert, D., and Weiss, C. (2020). The growing digital divide in europe and the united states. *European Investment Bank Working Paper*.

Weiss, J. (2019). Intangible investment and market concentration. *Working Paper*.

'Market Power and Innovation in the Intangible Economy'
Appendix - For Online Publication Only

Appendix A. Proofs and Derivations

Derivation of positive derivative in Section 2.1

The first order condition for intangibles implies that firms with lower adoption costs (higher ϕ) choose to reduce their marginal costs by a greater fraction s_i. To show that these firms also have a higher share of fixed (intangible) costs in total costs I prove that the latter increases in the fraction of marginal costs automated (s_{it}). Define b_{it} as the log of the share and take the derivative with respect to s_{it}:

$$\frac{\partial\, b_{it}}{\partial\, s_{it}} = \frac{\partial f(s_{it},\phi_i)/\partial s_{it}}{f(s_{it},\phi_i)} - \frac{\partial f(s_{it},\phi_i)/\partial s_{it} + (1-s_{it})\cdot \boldsymbol{c}(..)\cdot(\partial y_{it}/\partial s_{it}) - y_{it}\cdot \boldsymbol{c}(..)}{f(s_{it},\phi_i) + (1-s_{it})\cdot \boldsymbol{c}(..)\cdot y_{it}}$$

Grouping terms yields:

$$\frac{\partial\, b_{it}}{\partial\, s_{it}} = \frac{\partial f(s_{it},\phi_i)}{\partial s_{it}} \cdot \left(f(s_{it},\phi_i)^{-1} - (f(s_{it},\phi_i) + (1-s_{it})\cdot \boldsymbol{c}(..)\cdot y_{it})^{-1}\right) + \frac{\boldsymbol{c}(..)\cdot\left[y_{it} - (1-s_{it})\cdot(\partial y_{it}/\partial s_{it})\right]}{f(s_{it},\phi_i) + (1-s_{it})\cdot \boldsymbol{c}(..)\cdot y_i}$$

All terms on the right hand side of this expression are positive, provided that $y_i \geq (1-s_{it})\cdot(\partial y_{it}/\partial s_{it})$. Given that $y_{it} = (1-s_{it})^{-1}\cdot z(z_{it,1}, z_{it,2}, .., z_{it,k})\cdot \omega_i^{-1}$, this condition can be written as:

$$z(z_{it,1}, z_{it,2}, .., z_{it,k}) \geq \frac{\partial z(z_{it,1}, z_{it,2}, .., z_{it,k})}{\partial s_{it}}$$

which is the condition set out in equation (2).

Proof of Proposition 1

The value function is given by the following Bellman equation:

$$rV_t(\phi_i,\tilde{J}_i) - \dot{V}_t(\phi_i,\tilde{J}_i) = \max_{x_i} \left\{ \begin{array}{c} \sum_{j\in\tilde{J}_i} \left[\begin{array}{c} \pi_t(\phi_i,\lambda_{ij}) + \\ \tau(\phi_i)\cdot\left[V_t(\phi_i,\tilde{J}_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i,\tilde{J}_i)\right] \end{array} \right] \\ +x_i\cdot \mathrm{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1\right)\cdot \mathbb{E}_{\phi_i}\left[V_t(\phi_i,\tilde{J}_i \cup_+ \lambda_{ij}) - V_t(\phi_i,\tilde{J}_i)\right] \\ -w_t\cdot\eta_x\cdot(x_i)^{\psi_x}\cdot n_i^{\sigma} - F(\phi_i,n_i) \end{array} \right\}$$

Guess that the solution takes the following form:

$$V_t(\phi_i,\tilde{J}_i) = \sum_{j\in\tilde{J}_i} v_t(\phi_i,\lambda_{ij})$$

where $v_t(\cdot)$ (and hence V_t) grows at a constant rate g in the balanced growth equilibrium. Then $v_t(\phi_i, \lambda_{ij})$ can be written as:

$$\left[r - g + \tau(\phi_i)\right] \cdot v_t(\phi_i, \lambda_{ij}) = \pi_t(\phi_i, \lambda_{ij}) + \Gamma$$

where Γ is the option value of innovation adjusted for the fixed term $F(\phi_i, n_i)$:

$$\Gamma = \max_{x_i} \left[\frac{x_i}{n_i} \cdot \text{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \mathbb{E}_{\phi_i}\left[v_t(\phi_i, \lambda_{ih}) \right] - w_t \cdot \eta_x \cdot (x_i)^{\psi_x} \cdot n_i^{\sigma - 1} \right] - \frac{F(\phi_i, n_i)}{n_i} \quad (1)$$

which is a function Γ. In order for the value function to scale with size along the guess (a simplification that is removed in Section 6), Γ must not change with the number of goods that the firm produces. I achieve that by choosing $F(\phi_i, n_i)$ such that $\Gamma = 0$. To find the $F(\phi_i, n_i)$ that achieves this, use that the first order condition satisfies:

$$\text{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \mathbb{E}_{\phi_i}\left[v_t(\phi_i, \lambda_{ih}) \right] = \psi^x \cdot w_t \cdot \eta_x \cdot (x_i)^{\psi_x - 1} \cdot n_i^{\sigma}$$

such that if $\Gamma = 0$, the fixed term satisfies:

$$F(\phi_i, n_i) = (\psi^x - 1) \cdot w_t \cdot \eta_x \cdot \left[x(\phi_i, n_i) \right]^{\psi_x} \cdot n_i^{\sigma}$$

With this constraint, optimal research and development expenditures satisfy the equation in Proposition 1:

$$x(\phi_i, n_i) = \left(\text{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \frac{\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)} \right]}{\eta^x \cdot \psi^x \cdot w_t} \right)^{\frac{1}{\psi^x - 1}} \cdot n_i^{\frac{\sigma}{\psi^x - 1}}$$

It follows that

$$V_t(\phi_i, \tilde{J}_i) = \frac{\sum_{j \in \tilde{J}_i} \pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)}$$

where operating profits satisfy:

$$\pi_t(\phi_i, \lambda_{ij}) = \left[1 - \frac{\left(\lambda_{ij} \cdot \frac{w}{Y} \cdot (1 - \phi_i)\right)^{\frac{1}{\psi + 1}}}{\lambda_{ij}} \right] \cdot Y - w \cdot (1 - \phi_i) \cdot \left(\left[\lambda_{ij} \cdot \frac{w}{Y} \cdot (1 - \phi_i) \right]^{\frac{-\psi}{\psi + 1}} - 1 \right)$$

which increases at rate g along the balanced growth path, confirming the initial guess.

Derivation of Aggregate Quantities and Proof of Proposition 2

The equilibrium wage is derived as follows. Start with the definition of aggregate output when each sector is in a betrand equilibrium:

$$\ln Y = \int_0^1 \int \mathbf{1}_{j\in f_i} \ln \left(q_{ij} \cdot y_{ij}\right) di\, dj$$

Inserting the firm's production function $y_{ij} = l_{ij}/(1-s_{ij})$ and demand function $y_{ij} = Y/p_{ij}$ yields:

$$\ln Y = \ln Y + \int_0^1 \int \mathbf{1}_{j\in \tilde{f}_i} \ln \left(q_{ij} \cdot (w \cdot [1-s_{ij}])^{-1} \cdot \mu_{ij}^{-1}\right) di\, dj$$

Isolating wage on the left hand side gives:

$$\ln w = \int_0^1 \int \mathbf{1}_{j\in f_i} \ln \left[\frac{q_{ij}}{1-s_{ij}}\right] di\, dj + \int_0^1 \int \mathbf{1}_{j\in f_i} \ln \left[\frac{1-s_{ij}}{1+\lambda_{ij}}\right] di\, dj$$

The derivation of GDP is as follows. Labor market equilibrium requires:

$$L^p = \int_0^1 \int \mathbf{1}_{j\in f_i} l_{ij} di\, dj$$

Inserting the firm's production function $y_{ij} = l_{ij}/(1-s_{ij})$ and demand function $y_{ij} = Y/p_{ij}$ yields:

$$L^p = \int_0^1 \int \mathbf{1}_{j\in f_i} Y \cdot p_{ij}^{-1} \cdot \left(1-s_{ij}\right) di\, dj$$

Isolate Y on the left hand side, insert the first order condition for pricing, and insert the equilibrium wage to obtain:

$$Y = L^p \cdot \exp\left(\int_0^1 \int \mathbf{1}_{j\in f_i} \ln \left[\frac{q_{ij}}{1-s_{ij}}\right] di\, dj\right) \cdot \frac{\exp \int_0^1 \int \mathbf{1}_{j\in \tilde{f}_i} \ln \mu_{ij}^{-1} di\, dj}{\int_0^1 \int \mathbf{1}_{j\in \tilde{f}_i} \mu_{ij}^{-1} di\, dj} \tag{2}$$

Define total factor productivity Q_t as the terms to the right of L^p in expression (2). A balanced growth path equilibrium is characterized by constant type-shares $K(\phi_i)$. Given that markups equation $\lambda_{ij}/(1-s_{ij})$ where s_{ij} is given by equation (13), the law of large numbers assures that the third term in (2) is constant. Hence $g = \partial \ln Q/\partial t$ is given by:

$$g = \int_0^1 \int \mathbf{1}_{j\in f_i} \frac{\partial \ln q_{ij}}{\partial t} di\, dj = \sum_{\phi_i \in \Phi} K(\phi_i) \cdot \tau(\phi_i) \cdot \mathbb{E}_{-\phi_i}(\lambda_{hj})$$

which uses that $K(\phi_i) \cdot \tau(\phi_i)$ is the fraction of goods that changes producer each instance and where initially produced by ϕ_i-type firms.

Proof of Proposition 3

The value function is given by the following Bellman equation:

$$rV_t(\phi_i,\tilde{J}_i)-\dot{V}_t(\phi_i,\tilde{J}_i)=\max_{x_i}\left\{\begin{array}{c}\sum_{j\in\tilde{J}_i}\left[\begin{array}{c}\pi_t(\phi_i,\lambda_{ij})+\\ \tau(\phi_i)\cdot\left[V_t(\phi_i,\tilde{J}_i\setminus\{\lambda_{ij}\})-V_t(\phi_i,\tilde{J}_i)\right]\end{array}\right]\\ +x_i\cdot\text{Prob}\left(\lambda_{ij}\geq\frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})}-1\right)\\ \cdot\mathbb{E}_{\phi_i}\left[V_t(\phi_i,\tilde{J}_i\cup_+\lambda_{ij})-V_t(\phi_i,\tilde{J}_i)\right]-w_t\cdot\eta_x\cdot(x_i)^{\psi_x}\cdot n_i^{-\sigma})\end{array}\right\}$$

Guess that the solution takes the following form:

$$V_t(\phi_i,\tilde{J}_i)=\sum_{j\in\tilde{J}_i}\Upsilon_t^1(\phi_i,\lambda_{ij})+\Upsilon_{t,n_i}^2(\phi_i)$$

where firm i produces a portfolio of goods $\tilde{J}_i$ with cardinality n_i, and where $\Upsilon_t^1(\cdot)$ and $\Upsilon_{t,n_i}^2(\cdot)$ (and hence V_t) grow at a constant rate g in the balanced growth equilibrium. Grouping terms yields:

$$(r-g+\tau(\phi_i))\cdot\Upsilon_t^1(\phi_i,\lambda_{ij})=\pi_t(\phi_i,\lambda_{ij})\Rightarrow\Upsilon_t^1(\phi_i,\lambda_{ij})=\frac{\pi_t(\phi_i,\lambda_{ij})}{r-g+\tau(\phi_i)}$$

The proof of proposition 1 showed that profits grow at rate g, confirming the guess. Furthermore:

$$\left(r-g\right)\cdot\Upsilon_{t,n_i}^2(\phi_i)=\max_{x_i}\left\{\begin{array}{c}n_i\cdot\tau(\phi_i)\cdot\left[\Upsilon_{t,n_i-1}^2(\phi_i)-\Upsilon_{t,n_i}^2(\phi_i)\right]+x_i\cdot\text{Prob}\left(\lambda_{ij}\geq\frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})}-1\right)\\ \cdot\mathbb{E}_{\phi_i}\left[\Upsilon_{t,n_i+1}^2(\phi_i)-\Upsilon_{t,n_i}^2(\phi_i)+\Upsilon_t^1(\phi_i,\lambda_{ij})\right]-w_t\cdot\eta_x\cdot(x_i)^{\psi_x}\cdot n_i^{-\sigma})\end{array}\right\}$$

The first order condition of the maximization reads:

$$\text{Prob}\left(\lambda_{ij}\geq\frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})}-1\right)\cdot\mathbb{E}_{\phi_i}\left[\Upsilon_{t,n_i+1}^2(\phi_i)-\Upsilon_{t,n_i}^2(\phi_i)+\Upsilon_t^1(\phi_i,\lambda_{ij})\right]=w_t\cdot\psi_x\cdot\eta_x(x_i)^{\psi_x-1}n_i^{-\sigma}$$

Inserting the first order condition and isolating $\Upsilon_{t,n_i+1}^2(\phi_i)$ and $\Upsilon_t^1(\phi_i,\lambda_{ij})$ on the left hand side gives the sequence for Υ_{t,n_i+1}^2 along:

$$\begin{aligned}\Upsilon_{t,n_i+1}^2(\phi_i)+\Upsilon_t^1(\phi_i,\lambda_{ij}) \quad = \quad & \left[\frac{(r-g)\cdot\Upsilon_{t,n_i}^2(\phi_i)+n_i\cdot\tau(\phi_i)\cdot\left[\Upsilon_{t,n_i}^2(\phi_i)-\Upsilon_{t,n_i-1}^2(\phi_i)\right]}{\psi^x-1}\right]^{\frac{\psi^x-1}{\psi^x}}\\ & \cdot\text{Prob}\left(\lambda_{ij}\geq\frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})}-1\right)^{-1}\cdot\psi^x\cdot(\eta\cdot w_t)^{\psi^x\ 1}\cdot n_i^{-\frac{\sigma}{\psi^x}}+\Upsilon_{t,n_i}^2(\phi_i).\end{aligned}$$

Appendix B. Data

B1. Construction of the French Administrative Dataset

Balance Sheet and Income Statement The main firm-level datasets are FICUS from 1994 to 2007 and FARE from 2008 to 2016. I keep all firms in legal category 5, which means all non-profit firms and private contractors are excluded from the sample. I also drop firms with operating subsidies in excess of 10% of revenues. From 2004, INSEE starts to group firms that are owned by the same company in single *siren* codes. This treatment has been gradually extended over time, which means that data on groups in later years of the data contain more consolidated firms. From 2009 onwards, data is provided separately for the underlying firms (legal entities) and for the group. To have a consistent panel (and prevent an artificial increase in firm concentration), I group firms along the pre-2009 definitions and extend that treatment backwards and forwards.

Software and IT Data on software comes from the Annual Enterprise Survey (*Enquête Annuelle d'Entreprises*, EAE), which is an annual survey of around 12,000 firms between 1994 and 2007. There are separate surveys for major industries (agriculture, construction, manufacturing, services, transportation) which differ in variables and coverage. The survey is comprehensive for firms with at least 20 employees, and smaller firms are sampled for all sectors except manufacturing. The survey is merged to FARE-FICUS using the SIREN firm identifier. The level of observation is the legal unit, for firms that are aggregated prior to 2009 by INSEE as discussed in the main text. From 2008 onwards I use data from the E-Commerce Survey (*Enquête sur les Technologies de l'Information de la Communication* - TIC). This survey contains questions on the use of IT systems annually from 2008 to 2016. This dataset contains dummies on the adoption of specific IT systems such as Enterprise Resource Planning and Customer Resource Management.

Research and Development Data on R&D comes from the Community Innovation Survey (*Enquête Communautaire sur L'Innovation* - CIS). The CIS is carried out by national statistical offices throughout the European Union, and is coordinated by Eurostat. The survey is voluntary, but sample weights are adjusted for non-response to create nationally representative data. The French survey is carried out by INSEE, and contains consistent variables on research and development expenditures in 1996, 2000, 2004, 2006, 2008, 2010, 2012, 2014 and 2016.

Product Count The number of products by firm comes from the Annual Production Survey (*Enquête Annuelle de Production*, EAP). This survey is used for annual data on industrial production for the EU's PRODCOM statistics. The survey is available for manufacturing only, from 2009 to 2016. I count the number of unique products each year by firm, excluding products on which the firm acts as outsourcer, or was only involved in product design (M1 and M5).

B2. Variable Definitions

Compustat Data

Revenue is total sales. The Compustat Fundamentals variable is SALE.
Cost of goods sold involves all direct costs involved with producing a good. This includes the cost of materials and other intermediate inputs, as well as the labor directly used to produce a good. It is observed on the income statement. The Compustat variable is COGS.
Selling, general and administrative expense are all direct and indirect selling, general and administrative expenses. They include overhead costs and costs such as advertisement or packaging and distribution. It is observed on the income statement. The Compustat variable is XSGA.
Operating expenses are the sum of cost of goods sold and selling, general, and administrative expenses. The Compustat variable is XOPR.
Capital stock The firm's production capital is defined as the contemporaneous balance sheet value of gross property, plants and equipment (tangible fixed assets). The Compustat variable is PPEGT.
Operating profits are measured as income before extraordinary items. I add expenditures on research and development because these are expensed in the American data yet not in the French data. This furthermore prevents a spuriously positive correlation between the fixed cost measure (which declines in profits) and research and development. The Compustat variable is IB.
Research and development expenditures include all the costs incurred for the development of new products and services. They also include R&D activities undertaken by others for which the firm paid. They are observed on the income statement. The Compustat variable is XRD.
Product count is obtained from the Compustat Historical Segments File. I count the number of products that firms produce as the number of unique primary 6-digit NAICS codes of business segments that firms report. In the adjusted product count I assign a product count of 1 for firms that are not present in the segments file.

French Administrative Data

Revenue is total sales, including exports. In FICUS years this is CATOTAL, in FARE years this is REDI_R310. In regressions, firm-size is controlled for by a third degree polynomial of log revenue.
Employment Employment is the full-time equivalent of the number of directly employed workers by the firm averaged over each accounting quarter. In FICUS, the data is based on tax records for small firms, and on a combination of survey and tax data for large firms (variable name: EFFSALM). In FARE the variable is REDI_E200, which is based on the administrative DADS dataset.
Wage bill The wage bill is defined as the sum of wage payments (SALTRAI in FICUS, REDI_R216 in FARE) and social security contributions (CHARSOC in FICUS, REDI_R217 in FARE).
Direct production inputs are calculated as the sum of merchandise purchases (goods intended for resale) and the purchase of raw materials, corrected for fluctuations in inventory. In FICUS, the respective variables are ACHAMAR, ACHAMPR, VARSTMA, and VARSTMP. The corresponding variables in FARE are REDI_R210, REDI_R212,REDI_R211, and REDI_213.

Other purchases Other purchases are defined as purchases of services form other firms. This includes outsourcing costs, lease payments, rental charges for equipment and furniture, maintenance expenses, insurance premiums, and costs for external market research, advertising, transportation, and external consultants (AUTACHA in FICUS, REDI_R214 in FARE).

Operating profits is defined as revenue minus the wage bill, expenditure on direct production inputs, other purchases, import duties and similar taxes (IMPOTAX in FICUS, REDI_R215 in FARE) capital depreciation (DOTAMOR in FICUS), provisions (DOTPROV in FICUS), and other charges (AUTCHEX in FICUS). The sum of the wage bill, material input expenses, capital depreciation, provisions, and other charges is REDI_R201 in FARE.

Capital stock Capital is measured as the stock of fixed tangible assets. This includes land, buildings, machinery, and other installations. The associated variable is IMMOCOR in FICUS, and IMMO_CORP in FARE. The capital stock is not calculated using the perpetual inventory method because investment data is unavailable for 2008.

Industry codes Industry codes are converted to NACE Rev. 2 codes using official nomenclatures. Firms that are observed before and after changes to industry classifications are assigned their NACE Rev. 2 code for all years, while other firms are assigned a code from official nomenclatures. Firms in industries without a 1-to-1 match in nomenclatures are assigned the NACE Rev. 2 that is observed most frequently for firms with their industry codes. Firms that switch industry codes are assigned their modal code for all years.

Research and Development R&D investments are measured as all innovative expenditures by firms as reported in the CIS. Subcategories of expenditures fluctuate with each version of the survey, but total expenditures seems consistently defined. In 2012 total expenditures are found in RALLX. In some year I add up underlying variables to create a similar variable. Details for each year are available upon request.

Software Investments The variable for software investments closely follows the definition in Lashkari et al. (2019). The underlying variables are observed from 1994 to 2007 in the EAE. The main variable for software is I460. This variable contains all software investments and is available for all sectors. Because missing observations are coded as 0, I drop these firm-years when analysing software. An additional sub-division into externally purchased and internally developed software is available for a subset of firms (I461, I462, I463, I464, I465). Where available, I use this to clean cases where I460 is smaller than I461-I465, and verify that summary statistics match Lashkari et al. (2019).

Appendix C. Markup and Fixed Costs Estimation

This appendix summarizes the implementation of the iterative GMM approach by De Loecker and Warzynski (2012) that is used to estimate the output elasticity of a variable input m in order to calculate markups for fixed costs along the equation in Section 2.2. The production function estimation relies on codes developed for Burstein et al. (2019) who analyse the cyclical properties of French markups, and I thank the authors for permission to use the code for this project. I first outline the estimation procedure of markups for both France and the U.S., and subsequent discuss the robustness of the resulting series for fixed costs. I also discuss the implication of recent criticisms on the method that I use to calculate markups.

C1. Estimation Procedure

France

Because equation (1) contains both tangible (through $z(\cdot)$) and intangible inputs (through s_i), the framework in Section 2.1 implies a production function along $\tilde{z}(z_{it,1},..,z_{it,k};u_{it,1},..,u_{it,h})\cdot\omega_{it}$ with k tangible and h intangible inputs, Hicks neutral productivity ω_{it}, and potentially increasing returns to scale. I approximate this general production function by estimating a flexible translog function that contains the (squared) log of all observed inputs. I first estimate a production function with capital k, labor l and materials m for each 2-digit industry with at least 12 firms in the data, along:

$$y_{it} = \beta^l \cdot l_{it} + \beta^{ll} \cdot l_{it}^2 + \beta^k \cdot k_{it} + \beta^{kk} \cdot k_{it}^2 + \beta^m \cdot m_{it} + \beta^{mm} \cdot m_{it}^2 + \omega_{it} + \epsilon_t \tag{3}$$

where cross-terms are omitted to prevent measurement error in one of the inputs to directly affect the estimated elasticity of other inputs.[40] Capital is measured through fixed tangible assets, labor is the number of employees and materials equal firm purchases. In contrast to (i.e.) U.S. Census data, data on materials is available annually for firms in all industries.

The three-factor production function is commonly used in the literature and is therefore the basis of estimates in the main text. To assess the robustness of these estimates, I also estimate a more extensive production function with four production factors. The FARE-FICUS dataset allows materials to be divided into direct production inputs v (intermediate goods for resale and expenses on primary commodities) and other purchases o, which include the purchase of external services like advertising. I estimate an additional production function that separates these logged factors along:

$$y_{it} = \beta^l \cdot l_{it} + \beta^{ll} \cdot l_{it}^2 + \beta^k \cdot k_{it} + \beta^{kk} \cdot k_{it}^2 + \beta^v \cdot v_{it} + \beta^{vv} \cdot v_{it}^2 + \beta^o \cdot o_{it} + \beta^{oo} \cdot o_{it}^2 + \omega_{it} + \epsilon_t \tag{4}$$

Because of the large number of firms in the data, I estimate this more extensive production function separately for each 4-digit industry.

[40] This follows De Loecker et al. (2020) in their treatment of capital.

All inputs but material are likely to be a combination of tangible and intangible inputs in the context of Section 2.1's model, with the exception of direct production inputs.[41] Direct production inputs are tangible, as they only include expenses on intermediate goods for resale or expenses on primary commodities. An output elasticity can only be used to estimate markups when the factor is freely set each period, which seems most likely to hold for v. That is why I use the elasticity of output with respect to v to estimate markups from the four-factor production function.

Both production functions are estimated under the assumption that a firm's demand for material is an invertible function $m(\cdot)$ (or $v(\cdot)$) of the firm's productivity ω_{it} and capital and labor inputs. As a consequence, the production functions can be written as:

$$y_{it} = \beta^l \cdot l_{it} + \beta^{ll} \cdot l_{it}^2 + \beta^k \cdot k_{it} + \beta^{kk} \cdot k_{it}^2 + \beta^m \cdot m_{it} + \beta^{mm} \cdot m_{it}^2 + m^{-1}(\omega_{it}, l_{it}, k_{it}) + \epsilon_t \text{ and}$$

$$y_{it} = \beta^l \cdot l_{it} + \beta^{ll} \cdot l_{it}^2 + \beta^k \cdot k_{it} + \beta^{kk} \cdot k_{it}^2 + \beta^v \cdot v_{it} + \beta^{vv} \cdot v_{it}^2 + \beta^o \cdot o_{it} + \beta^{oo} \cdot o_{it}^2 + v^{-1}(\omega_{it}, l_{it}, k_{it}) + \epsilon_t$$

respectively. Under this assumption, I purge log gross output y_{it} from measurement error by estimating:

$$y_{it} = h(l_{it}, k_{it}, m_{it}) + \varepsilon_{it} \text{ and } y_{it} = h(l_{it}, k_{it}, v_{it}, o_{it}) + \varepsilon_{it}$$

where h is a non-parametric function approximated by a third degree polynomial in the inputs.

After purging gross output, the production function is estimated iteratively. The algorithm is as follows. First, I guess the coefficients of the production function using OLS estimates. Given (purged) output, inputs, and the production function, I calculate ω_{it}. The algorithm then estimates the autoregressive process of productivity along:

$$\omega_{i,t} = g' \left[1 \ \omega_{i,t-1} \ \omega_{i,t-1}^2\right]' + \xi_{i,t}$$

where residual ξ_{it} captures shocks to productivity not explained by (squared) lagged values of productivity, while g is a vector of coefficients obtained by minimizing the sum of squared residuals $\xi_{i,t}$:

$$g = \left(\begin{bmatrix} 1 & \omega_{t-1} & \omega_{t-1}^{\circ 2} \end{bmatrix} \begin{bmatrix} 1 \\ \omega_{t-1} \\ \omega_{t-1}^{\circ 2} \end{bmatrix} \right)' \left(\begin{bmatrix} 1 & \omega_{t-1} & \omega_{t-1}^{\circ 2} \end{bmatrix} \omega_t \right) \tag{5}$$

The algorithm iterates the production function coefficients until the errors of the AR(1) process for productivity satisfy:

$$\mathbb{E}\left(\xi_{it} Z_{i,t}\right) = 0 \tag{6}$$

where $Z_{i,t}$ is a vector of instruments:

$$Z_{i,t} = \left[l_{it-1} \ l_{it-1}^2 \ k_{it} \ k_{it}^2 \ m_{it-1} \ m_{it-1}^2\right]'$$

[41] Labor may seem a tangible input, but if labor is used to develop or deploy software for production then the intangible input labor appears on the income statement through the wage bill.

or for the four-factor production function:

$$Z_{i,t} = \left[l_{it-1} \;\; l^2_{it-1} \;\; k_{it} \;\; k^2_{it} \;\; v_{it-1} \;\; v^2_{it-1} \;\; o_{it-1} \;\; o^2_{it-1} \right]'$$

By instrumenting k with its current value, I assume that firms cannot increase capital in response to a contemporaneous productivity shock. By instrumenting l, m, v and o by their lagged value I assume that they are set freely each period, but require autocorrelation in factor prices.[42]

Gross output in the production function is measured through sales, which has been criticized in a number of recent papers. While a review of the debate goes beyond the scope of this paper, a particularly relevant critique is presented in Bond et al. (2020). They show that when markups are measured by multiplying the inverse of a factor's share in revenue with the revenue function elasticity rather than the production function elasticity, the resulting markup is biased in such a way that its value should always equal 1.

In practice, markups estimated with the De Loecker and Warzynski (2012) methodology do not measure the revenue elasticity as revenue is purged from factors unrelated to input usage in the first stage. The French data furthermore allows for a comparison of markups obtained from data on revenue versus data on quantities, because the French product-level data on manufacturing (the *EAP*) contains price data. Using this data, Burstein et al. (2019) show that the firm-level markups based on quantity data have a 0.83 correlation coefficient with markups based on revenue data. Note, furthermore, that the model only relies on fixed costs in order to calibrate the initial level of intangible efficiency. Bias in markup estimates therefore only affect the initial calibration of ϕ_i.

United States

To estimate markups for the calculation of fixed costs of U.S. publicly listed firms I deploy the same procedure. A constraint of the analysis of markups for these firms is that data on materials and the wage bill is not available from the income statement. Instead, there is a broad category of operating expenses (cost of goods sold) that captures all expenditures that are directly related to the cost of production. This is the variable used for flexible inputs in De Loecker et al. (2020), whose procedure I follow closely. Results in the main text are based on a fixed cost measure that uses these markup estimates.

One critique on using a production function estimation with capital and cost of goods sold is that it does not account for selling, general, and administrative expenses (SG&A), which have become more important over time. Adding SG&A to cost of goods sold to form a single input in a production function is evenly problematic because 1) a large part of SG&A are fixed overhead costs as well as expenditures on intangible inputs,[43] and 2) this assumes that all types of operating expenses are perfect substitutes. Instead, I test the robustness of my main results by adding SG&A as a separate input in a production function along (3).

[42]For France it is reasonable to assume that labor is, in fact, not set freely and could therefore be instrumented by contemporaneously. This turns out to have no significant effect on the estimated production function.

[43]Heterogeneity in fixed costs across firms will then cause an underestimation of the input elasticities and markups.

Table A1: Summary Statistics on Estimated Markups

	Mean	Std. Dev.	Median	10th Pct.	90th Pct.	Observations
France						
Basic production function	1.38	0.43	1.26	0.96	1.91	9,913,058
Extended production function	1.42	1.25	1.01	0.53	2.59	8,477,467
United States						
COGS production function	1.52	.620	1.33	1.01	2.27	125,231
COGS and SG&A production function	1.33	.589	1.15	0.86	2.02	125,231

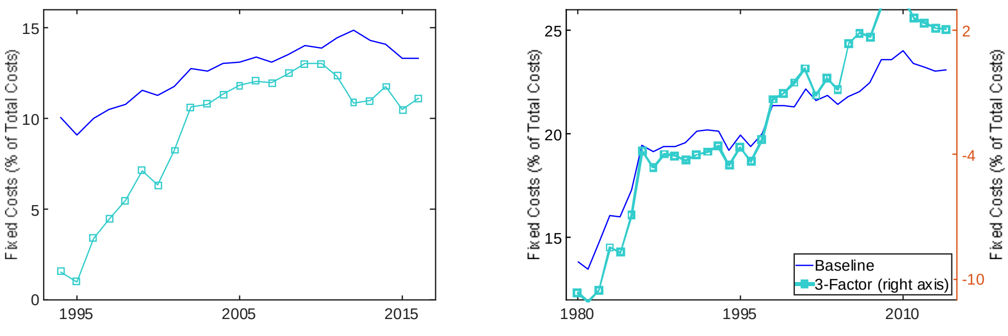
C2. Robustness of Fixed Cost Trends

France

The results in the main text are robust to using the more extensive four-factor production function. After estimating the industry-level production function coefficients, I calculate the firm-level markup as the product of the input elasticity and the inverse of the input's revenue share. I then calculate the fixed cost share along (3). Markups at the firm-level are summarized in Table A1. The table shows that the extensive production function estimates a very similar average markup to the markup from the standard three-factor production function. The variance of markups, however, is significantly greater when using the four-factor production function. This is likely due to the additional parameters that need to be estimated at the 4-digit level, or because firms have some flexibility in what costs fall under direct production inputs v versus other purchases o. The firm-level correlation coefficient between both markups is 0.35.

The trends of aggregate fixed costs are plotted in Figure A1. The solid-blue line is replicated from the main text and is for the three-factor standard production function, while the squared-green line uses the four-factor extensive production function. Both figures show that the sales-weighted average fixed cost share has increased strongly over the 1994 to 2016 sample, with the largest increase occurring between 1994 and 2010, after which the increase moderates.

Figure A1. Robustness of Trends in Aggregate Fixed Cost Share



(a) 3-Factor Production Function (b) United States

United States

Markups from the two-factor and three-factor production functions are highly correlated. The bottom panel of TableA1 presents summary statistics for both and shows that they mainly differ in terms of their their level. When adding SG&A, over 30% of all firms have markups below 1 and the median markup is 1.15. Though the 2-factor admits markups around 15 percentage points above that at most percentiles, both series co-move strongly. The firm-level correlation is 0.92. While the correlation of the markup series is close, the difference in levels between the series have a large effect on the predicted level of fixed costs. The right plot in Figure A1 shows that the 3-factor production function predicts *negative* average fixed costs as a percentage of total costs between 1980 and 2004. This is likely to be driven by an underestimation of the markup; of the firms with a 3-factor markup below unity, 63% report positive profits. The predicted increase in fixed costs over the sample is 13 percentage points, which is similar to the predicted increase in the main text.[44]

C4. Within versus Between Sector Changes in Rise of Fixed Costs

Figure A2 illustrates the sectoral composition of fixed costs. It shows that fixed costs as a fraction of total costs are especially high in the information sector (NAICS industry 51 for the U.S. and NACE industry JB and JC for France). The distribution of fixed costs across sectors is similar for the U.S. and France and the majority of sectors have seen an increase in their average ratio of fixed- to variable costs. The latter suggests that fixed costs have increased at the aggregate level because of an increase in the importance of fixed costs within sectors and not because high-fixed costs sectors have become larger over time. To formally show that the aggregate rise of fixed costs is driven by within-sector reallocation, I perform the following within-between decomposition:

$$\Delta \frac{\tilde{F}_t}{TC_t} = \sum_{j \in J} s_{jt-1} \cdot \Delta \frac{\tilde{F}_{jt}}{TC_{jt}} + \sum_{j \in J} \Delta s_{jt} \cdot \frac{\tilde{F}_{jt-1}}{TC_{jt-1}} + \sum_{j \in J} \Delta s_{jt-1} \cdot \Delta \frac{\tilde{F}_{jt}}{TC_{jt}}$$

where $\tilde{F}_t / TC_t$ is the aggregate fixed cost share, $\tilde{F}_{jt} / TC_{jt}$ the sector-level counterpart, and s_j the fraction of sales by sector j. The first term captures changes due to increases in fixed costs within sectors. The second term captures the 'between' share: changes because of changes in the relative

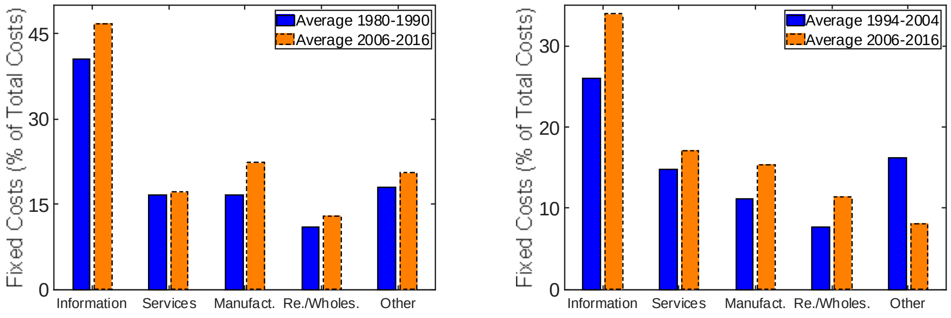
Table A2: Decomposition of Changes in Aggregate Fixed Cost Share

	Within Sectors	Between Sectors	Cross Term	Total
United States	1.02***	0.00	-0.02	1
	(0.053)	(0.050)	(0.015)	
France	0.73***	0.21***	0.06***	1
	(0.003)	(0.003)	(0.003)	

Standard errors in brackets. *** denotes significance at the 1% level.

[44] Figure A1 does raise concerns about the correct calibration target for the initial level of fixed costs. The baseline calibration uses 12%. De Loecker et al. (2020) assume that SG&A find that the of fixed costs has increased from 18% to 24% for Compustat firms. In unpublished work, Saibene (2017) finds that the share of fixed costs and total costs from 10% to 20% for Compustat firms, based on the sensitivity of costs to sales shocks. I conclude that the 12% calibration target for 1980 is within the plausible range of estimates.

Figure A2. Weighted-Average Ratio of Fixed Costs to Total Costs across Sectors



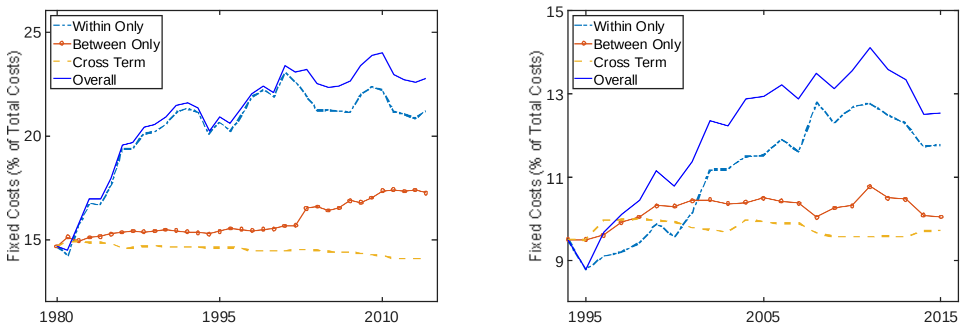
(a) **United States** (b) **France**

Notes: Sales-weighted average of fixed costs fraction by sector for U.S. listed firms (left) and the universe of French firms (right). Sectors are ordered by the average fixed-cost share in the last ten years of the French sample. Industry definitions for the United States (NAICS): 51 for information, 64 and above for services, 31, 32 for manufacturing, and 42, 44, 45 for wholesale and retail; for France (NACE/ISIC): JB, JC for information, I, M, N for services, B, C, D, E for manufacturing, and G for wholesale and retail.

size of sectors. The last term is the interaction of both. I perform the decomposition annually and regress each term on the change in the aggregate fixed cost share.

The resulting coefficients are presented in Table A2. Figure A3 illustrates the contribution of the within and between share over time, by plotting the development of fixed costs holding other contributors constant. The results show that within-sector reallocation was largely responsible for the rise of fixed costs, in both countries.

Figure A3. Within-Between Decomposition of the Rise of Fixed Costs



(a) **United States** (b) **France**

Notes: Within-between decomposition of the rise of fixed costs for U.S. listed firms (left) and the universe of French firms (right).

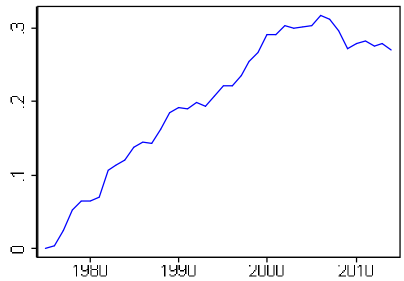
Appendix D. Macroeconomic Trends in France

The introduction summarizes three recent trends: the slowdown of productivity growth, the fall in business dynamism and the rise of corporate profits. This appendix gives an overview of the macroeconomic trends for France.[45]

The slowdown of productivity growth is depicted in Figure A4. It plots an index of the log of TFP at constant prices, standardized to 0 in 1975. The figure shows that TFP was growing at a steady rate for most years between 1975 and 2000. There was a significant slowdown in the early 2000s, and productivity growth over the 2005-2020 era has been slightly negative.

The decline in business dynamism is summarized with three statistics, following the literature. The first is the reallocation rate in Figure A5a, which is the sum of job destruction and creation rates. I calculate the reallocation rate across French firms using the FARE-FICUS dataset for 1994-2016. Because this sample coincides with the Great Recession, which brought a strong transitory increase in reallocation due to job destruction, I plot the HP trend. The second fact is the decline of entry of new firms. Figure A5b captures this trend by plotting the fraction of employees that work for a firm that enters the FARE-FICUS dataset in a given year. Note that this may include firms that have undergone significant organizational changes that have caused their firm identifier to change. The figure shows that employment by entrants has declined by almost half within the 1994-2016 sample. The third fact is the decline of skewness of the firm growth distribution. As discussed by Decker et al. (2017), small (young) high-growth firms have historically been an important contributor to productivity growth. They infer the decline in skewness of the growth distribution from the decline between the 90th and 10th, and between the 90th and 50th percentile of the growth distribution. Figure A6 shows that both have declined by around 40% between 1994-2016. The difference between the 50th and 10th percentile has remained flat, in line with U.S. evidence.

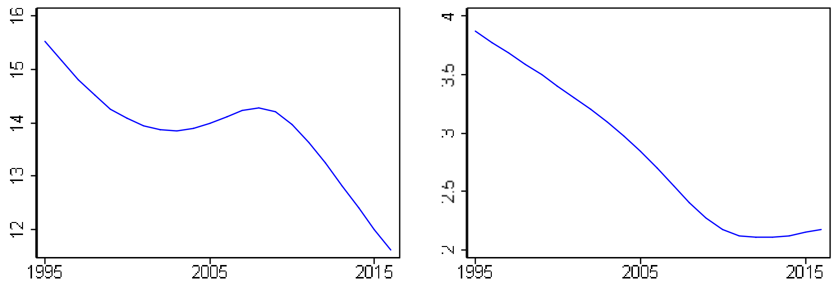
Figure A4. Total Factor Productivity in France



Log TFP at constant prices, 1975=0. Data: Penn World Tables.

[45]Whether market power is increasing across advanced economies remains a subject of debate. The slowdown of productivity growth and the decline of start-ups have been widely documented (e.g. Adler et al. 2017 and Calvino et al. 2016), while the rise of market power and firm concentration seems to be larger in the U.S. Döttling et al. (2017) and Cavalleri et al. (2019) find no increase in industry concentration in Europe between 2000 and 2013, using Orbis data. Bajgar et al. (2019) document a rise in concentration in most of Europe when accounting for ownership structures and the coverage of small firms in Orbis. Aquilante et al. (2019) also find an increase in U.K. industry concentration between 1998 and 2016.

Figure A5. Business Dynamism in France

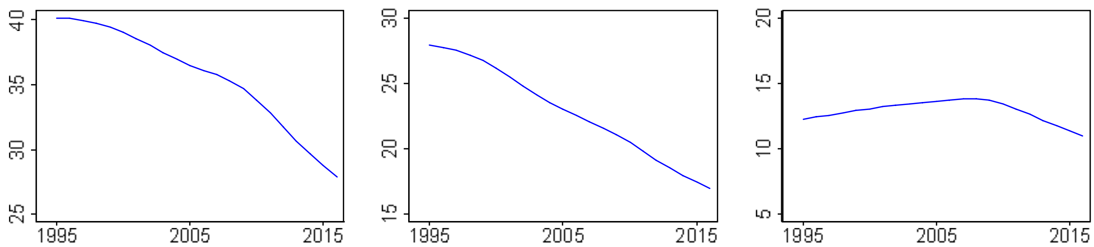


(a) **Reallocation Rate** (b) **Entrants**

Both figures plot HP trends. Left figure: sum of job creation and job destruction rates across companies. Right figure: Percentage of employment by new firms (≤ 1yr) in private sector employment. HP trend.

The rise of corporate profits is measured through the marginal cost markup. This is a measure of marginal rather than average profits, a distinction that is key in Section 2. Figure A7a plots the average sales-weighted markups for French firms between 1994 and 2016. The markups has increased modestly, in line with previous evidence (e.g. IMF 2019). Though not directly measuring market power, concentration also displays a modestly positive trend over the sample. This is shown in Figure A7b, which depicts the average Herfindahl Index across 5-digit industries. The rise of concentration has been linked to the decline in the labor share by Autor et al. (2020) through the reallocation of activity to firms with low labor shares. This result has been replicated for France for 1994-2007 by Lashkari et al. (2019). Note that the increase in concentration depends on measurement. The graph below presents an average of the Herfindahl across sectors. Weighing sectors by value added gives an increase in the Herfindahl index from 2008 from 0.087 in 1994 up to 0.122 in 2008, but a modest decline to 0.117 afterwards.

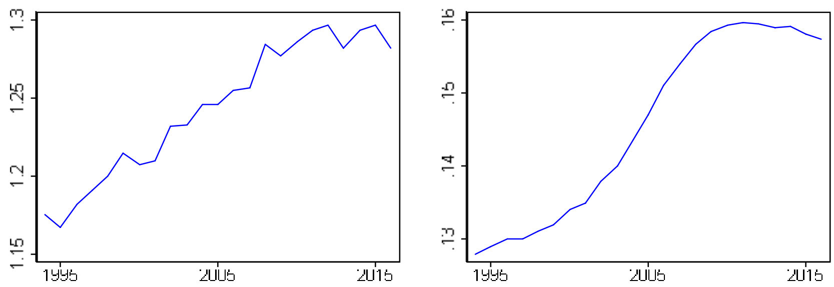
Figure A6. Skewness of the Employment-Growth Distribution



(a) **90-10 Difference** (b) **90-50 Difference** (c) **50-10 Difference**

Difference (perc. point) in growth between percentiles of the employment-growth distribution. HP trend.

Figure A7. Markups and Firm Concentration in France



(a) Markups (b) Concentration

Left figure: sales-weighted marginal cost markups using the Hall (1988) equation with production function elasticities estimated with iterative GMM as in De Loecker and Warzynski (2012). Details in Appendix C. Right figure: average Herfindahl index across 5-digit NACE industries. HP trend.

Appendix E. Computational Algorithm

The balanced growth path equilibrium along definition 1 is found by solving the system of de-trended equilibrium equations as a fixed point. The algorithm works as follows:

1. Solve the fixed point:
   (a) Guess a level of Y/Q, w/Q, $\tau(\phi)$, and $K(\phi)$.
   (b) Collect choke prices by solving:

$$\left(p^{choke}(\phi_i) - w \cdot [1 - s^*(\phi_i)]\right) \cdot Y - w \cdot (1 - \phi_i) \cdot \left([1 - s(\phi_i)]^{-\psi} - 1\right) = 0 \text{ where } \phi_i \in \Phi$$

   (c) Given the vector of choke prices and the guess for $K(\phi)$, calculate the following objects:
      - a $|\Phi| \times |\Phi|$ matrix $\mathbf{P}$ with probabilities that a firm of type $\phi_i \in \Phi$ successfully innovates when facing $\phi_{-i} \in \Phi$ along (11) and a vector with the weighted average over this probability $\sum_{\phi_{-i} \in \Phi} K(\phi_{-i}) \cdot \mathbf{P}(\phi_i, \phi_{-i})$ with the probabilities that a type's innovation is successful in general.
      - the set of distributions of $\lambda_{ij} \sim Exp(\bar{\lambda})$ for each combination of $\phi_i \in \Phi$ and $\phi_{-i} \in \Phi$ truncated at $p^{choke}(\phi_i)/p^{choke}(\phi_{-i})$.
      - the expectation of markups along (12) given the truncated distributions and the guess for $K(\phi)$.
      - the optimal innovation efforts by incumbents and entrants given markups, $\mathbf{P}$, Y, w, $\tau(\phi)$, and $K(\phi)$.

   (d) Calculate Y along (21) and w along (20). Use the innovation effort by incumbents and entrants to calculate $\tau(\phi)$ along (10) and (17), (18) and (19) to find $K(\phi)$.
   (e) Repeat from step (b) until the model has converged.

2. Perform the firm simulation:

   (a) Collect the equilibrium Y, w, $\tau(\phi)$, $K(\phi)$, $x(\phi,n)$, e for all n and all $\phi_i \in \Phi$.

   (b) Discretize time by introducing a sufficiently large number of instances per year such that $x(\phi,n) < 1$ and $e < 1$.

   (c) Initialize the firm-size distribution along (17) and (18).

   (d) Simulate firms until the markup distribution has converged, then collect moments.

The transitional dynamics are numerically solved using the following algorithm:

1. Create a fine grid with a T-year horizon, allowing each year to consist of $\tilde{T}$ instances.

2. Guess an initial value function of innovation activities $V(\cdot)$ equal to the new steady-state level for each type in $\phi_i \in \Phi$ at each point of the grid. Similarly guess the paths of wages w/Q and output Y/Q at their new steady-state level.

3. Initialize the firm-size and type distribution $K(\phi)$ and $M(\phi,n)$ to their original steady state.

4. Iterate over the path of the value function as follows:

   (a) Solve the static optimization problem and the dynamic innovation decisions for incumbents and entrants for each point on the grid using the initial guess for $V(\cdot)$.

   (b) Given the innovation and static decisions, simulate the development for a large (N) number of products and track the innovation step-sizes λ in $N \times (T \cdot \tilde{T})$ matrix Λ and similarly a matrix of ownership types using a forward loop over the grid.[46]

   (c) Update the value function using the new sequences for Y, w, the firm-type and -size distribution, and distributions for markups and λs implied by Λ. This involves calculating:

      i. the expectation of profits $\pi_{kt}(\phi_i, \lambda_{ij})$ at each instance t on the grid $t = 1,..,T$ separately for each cohort of patents k.

      ii. the value of obtaining the patent to produce an additional product for incumbents of type ϕ at time k as follows:

$$V^k(\phi_i) = \mathbb{E}^k_{\phi_i}\left[\sum_{t=k+1}^{\varepsilon \cdot T} \prod_{h=k+1}^{t} \left(\frac{1-\tau_h(\phi_i)}{1+\rho}\right) \cdot \pi_{kt}(\phi_i, \lambda_{ij})\right]$$

      which is a discretization of the original value function, where ϵ is set such that the present value of profits in instances exceeding $\epsilon \cdot T$ approaches zero.[47]

   (d) Use the resulting value for each type on each point of the grid as the guess for $V(\cdot)$ in step (a) in the next iteration. Continue until the path of the value function converges.

[46] This simulation is needed because the changing composition of firm types means the distribution of realized λs has no analytical representation. I then use the resulting distribution of markups to calculate the efficiency wedge along (21), as well as a path for Y and w. These serve as the basis for the algorithm's next iteration.

[47] I set $T = 3000$ (corresponding to 60 years), $\epsilon = 11$ (a profit horizon of 600 years), and set $N = 10000$.

Appendix F. Additional Figures and Tables

Table A3: Comparison of Theory and Data for Untargeted Moments

		United States			France		
	Quartile	**Model**	**Data**	**St. Dev.**	**Model**	**Data**	**St. Dev.**
Size and Age	1st (Age)	1.17	2.17	(1.04)	1.25	1.98	(1.01)
	2nd (Age)	1.46	2.28	(1.05)	1.68	2.39	(1.06)
	3rd (Age)	1.69	2.47	(1.09)	2.02	2.69	(1.07)
	4th (Age)	1.86	3.05	(1.08)	2.21	3.04	(1.03)
Exit Rate and Age	1st (Age)	.146	.114	(.318)	.131	.060	(.238)
	2nd (Age)	.133	.122	(.317)	.107	.055	(.229)
	3rd (Age)	.123	.110	(.306)	.090	.038	(.190)
	4th (Age)	.117	.075	(.265)	.080	.036	(.189)
Exit Rate and Size	1st (Size)	.153	.127	(.333)	.146	.114	(.318)
	2nd (Size)	.153	.109	(.312)	.146	.040	(.196)
	3rd (Size)	.153	.091	(.287)	.024	.028	(.165)
	4th (Size)	.023	.067	(.251)	.003	.024	(.153)
Product Loss Probability and Age	1st (Age)	0.161	.045	(.208)	.163	.105	(.306)
	2nd (Age)	0.178	.048	(.213)	.193	.127	(.333)
	3rd (Age)	0.193	.055	(.228)	.225	.152	(.359)
	4th (Age)	0.207	.068	(.252)	.242	.164	(.370)

Notes: U.S. data is from Compustat data (1980 to 2016). French data is from the full FICUS-FARE dataset (1994-2016). Size is measured as sector-deflated sales, age as the number of years since creation or Compustat entry. Exit is a dummy equal to 1 if a firm no longer appears in Compustat/FICUS-FARE in subsequent years. Product loss is a dummy equal to 1 if a firm produces fewer goods the subsequent year in the segment/EAP data. Items under 'model' and 'data' are the mean of the variable within the quartile considered.

Table A4: Balanced Growth Path Comparison - Robustness Check with $\psi = 0.86$

	United States			France		
	Δ Model (Main, $\psi = 2$)	**Δ Model** ($\psi = 0.86$)	**Δ Data**	**Δ Model** (Main, $\psi = 2$)	**Δ Model** ($\psi = 0.86$)	**Δ Data**
Cost Structure						
Average Fixed-Cost Share	3.8 pp	3.7 pp	10.6 pp	5.2 pp	4.1 pp	4.5 pp
Intangibles over Value Added	1.5 pp	2.1 pp	2.1 pp	3.6 pp	2.4 pp	2.2 pp
Slowdown of Productivity Growth						
Productivity Growth Rate	-0.43 pp	-0.44 pp	-0.9 pp	-0.23 pp	-0.36 pp	-1.3 pp
Aggregate R&D over Value Added	41.9%	33.2%	64.5%	67.2%	36.3%	5.6%
Decline of Business Dynamism						
Reallocation Rate	-42.0%	-43.4%	-23%	-23.8%	-35.8%	-23%
Entry rate	-5.8 pp	-5.8 pp	-5.8 pp	-4.5 pp	-4.5 pp	-4.5 pp
Rise of Market Power						
Average Markup	21.8 pt	21.2 pt	30 pt	22.9 pt	22.2 pt	11 pt
Model Objects						
Labor Wedge	8.8 pt	7.7 pt	N.A.	11.2 pt	8.2 pt	N.A.
Efficiency Wedge	.03 pt	.05 pt	N.A.	.02 pt	0.04 pt	N.A.

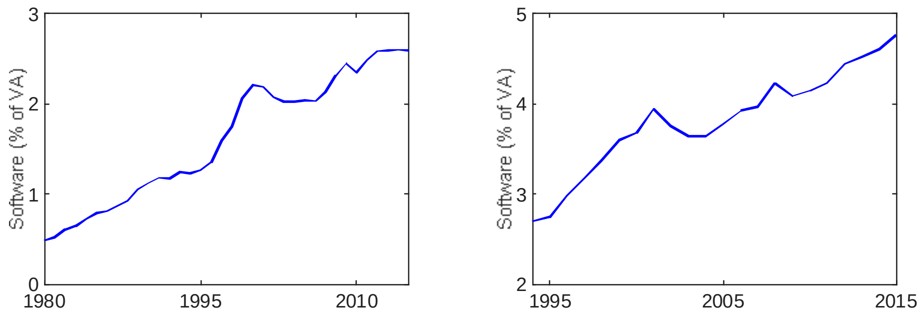
Notes: This table contains a robustness check for the balanced growth path results in Table 6. Rather than estimating the model with $\psi = 2$, the model is estimated with $\psi = 0.86$. This achieves a pass-through of marginal cost shocks to markups of -35% rather than -25%, in line with the main results in Mian et al. (2013). Data columns present the empirical moments, while model columns present the theoretical moments. The change in productivity growth is the difference between growth from 1969-1979 (U.S.) or 1969-1994 (France) to growth post-2005. Other U.S. moments equal the difference between 1980 and 2016. Other French moments equal the difference between 1994 and in 2016.

Table A5: Structural Estimation - Alternative Value Function Specification

Par.	Moment	United States				France			
		Par. Value (Old/New)	Data Target	Model (Main)	Model (Full)	Par. Value (Old/New)	Data Target	Model (Main)	Model (Full)
η^x	R&D Intensity	3.95/3.95	2.5%	2.4%	2.5%	2.15/2.15	3.1%	2.6%	3.3%
η^e	Entry Rate	2.98/2.98	13.8%	13.5%	14.1%	3.27/3.27	10.0%	9.9%	9.6%
$\bar{\lambda}$	Productivity Gr.	0.07/0.07	1.3%	1.3%	1.3%	0.07/0.07	1.3%	1.3%	1.4%
σ	Gibrat's Law	0.47/0.52	-0.035	-0.035	-0.035	0.60/0.67	-0.035	-0.035	-0.035
ϕ	Fixed Costs (%)	0.81/0.81	13.9%	14.2%	14.1%	0.74/0.74	9.5%	9.5%	10.2%

Notes: Data columns present the empirical moments while model columns present the theoretical moments. U.S. moments are for 1980 except for the regression coefficient of firm growth on firm size, which is taken from Akcigit and Kerr (2018). French moments are for 1994 or the first subsequent year for which the moment is present in the micro data.

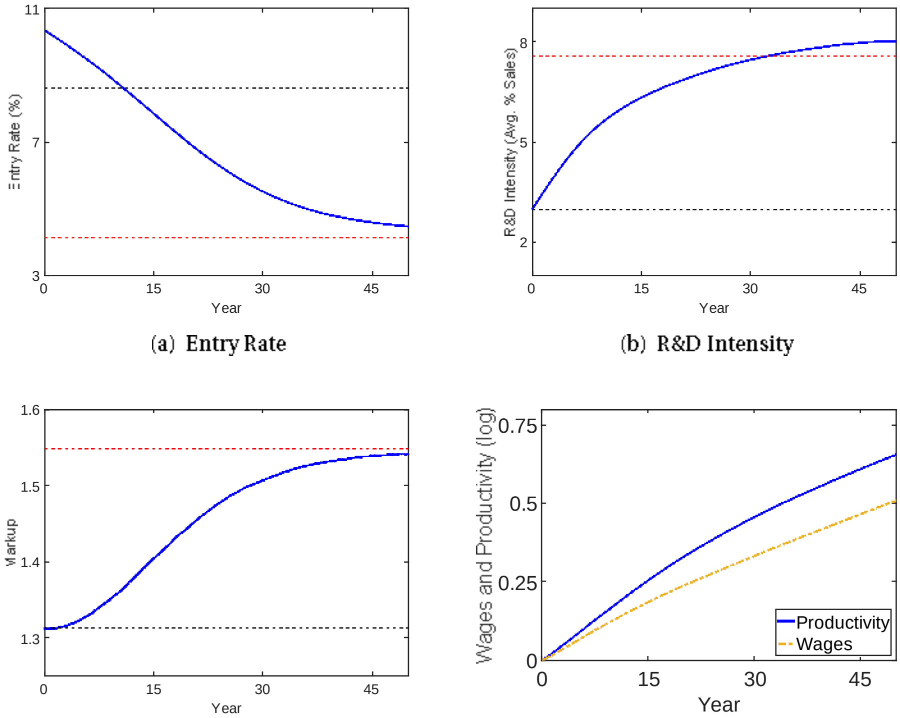
Figure A8. Software Investments as a Percentage of Value Added over Time



(a) **United States** (b) **France**

Notes: Investments in software as a percentage of private sector value added. U.S. data is obtained from the BEA NIPA tables. French data, which includes database investments, is from EU KLEMS. Time windows match calibration targets.

Figure A9. Transition Path for Various Variables (France)



(a) **Entry Rate**

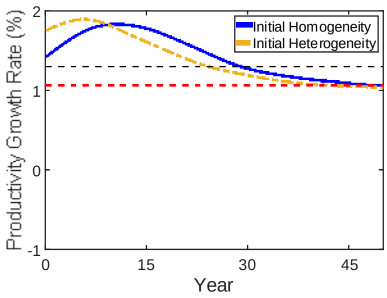
(b) **R&D Intensity**

(c) **Markups**

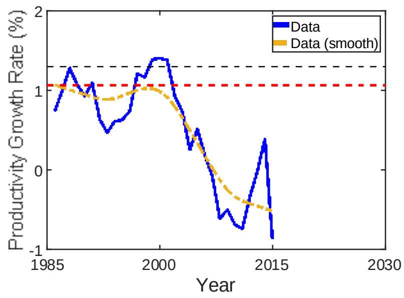
(d) **Wages and Productivity**

Black and red dashed lines (respectively) indicate the original and the new steady state. Figure (a) presents the entry rate, (b) presents R&D intensity (the average ratio of R&D over sales), (c) presents the average markup, (d) presents the path of wages (which tracks quality) and productivity (which tracks quality and intangibles).

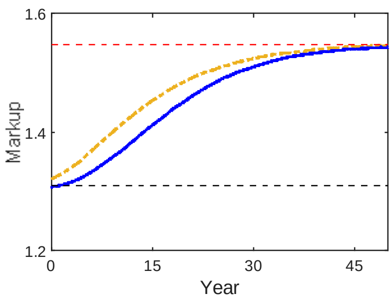
Figure A10. Transition Path: Model Predictions versus Data (France)



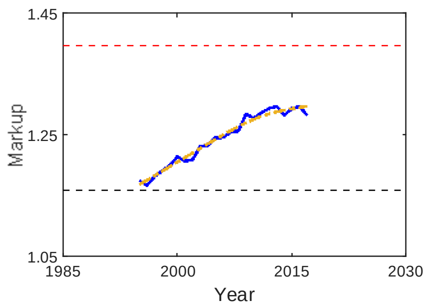
(a) **Productivity Growth (Model)**



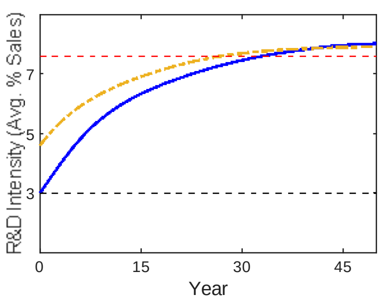
(b) **Productivity Growth (Data)**



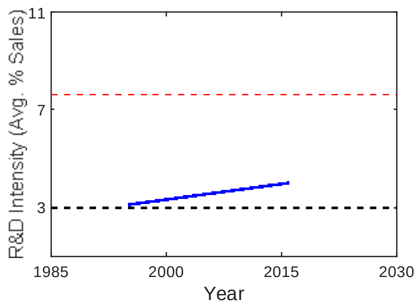
(c) **Markup (Model)**



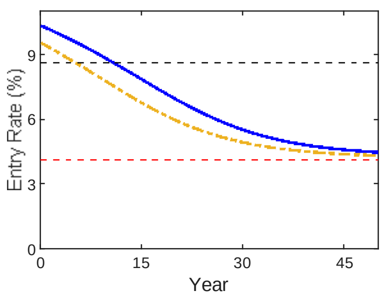
(d) **Markup (Data)***



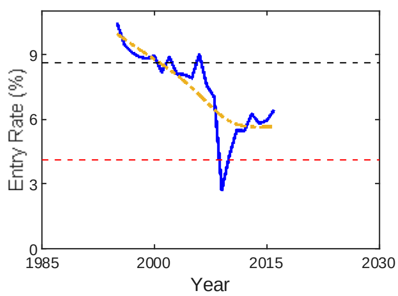
(e) **R&D Intensity (Model)**



(f) **R&D Intensity (Data)***



(g) **Entry Rate (Model)**



(h) **Entry Rate (Data)**

Black and red dashed lines (respectively) indicate the original and the new steady state. Calibration is for France. Productivity growth in Figure 8b is a 5-year centred moving average to reduce noise. HP-filter smoothing parameter is 100. Data sources: productivity growth from Penn World Tables, R&D from CIS (1996, 2016), entry (imputed) and markups from FICUS-FARE.

* The axis for the markup data is re-scaled by subtracting 0.25 from the model's original and final steady state. This is because the initial level of the markup is untargeted, and the model's markup is 0.25 lower than the empirical markup in the original steady state.

Appendix G. Conditional Correlations: Fixed Costs, R&D and Growth

The model in Section 3 predicts a positive correlation between fixed costs, markups and research and development (R&D) at the firm level. This appendix shows that these relationships hold in the data by verifying that (conditional) correlations run in the appropriate direction in the French and the U.S. data from Section 2, with the cautionary remark that this does not imply causality.

To measure the correlation between fixed costs and R&D for French firms, I use data from the *Enquête Communautaire sur L'Innovation* (CIS). The CIS was held in 1996 and 2000, and biannually since 2004. The main variable from this dataset is expenditures on R&D, including externally purchased R&D and expenditures on external knowledge or innovation-related capital expenditures. For Compustat firms, I use R&D from the income statement (*xrd*).[48] The estimation equation reads

$$\frac{rd_{it}}{p_{it} \cdot y_{it}} = \alpha_i + \psi_t + \gamma \cdot \frac{\tilde{f}_{it}}{tc_{it}} + \beta' g(p_{it} \cdot y_{it}) + \varepsilon_{ijt}, \tag{7}$$

where R&D intensity is the dependent variable, as is standard in the literature (e.g. Hall et al. 2010). Results are presented in Table A6. The upper panel represents results for the French survey data, while the bottom panel presents results for the U.S. data. Upon adding firm fixed effects (columns III and IV), the tables present similar coefficients: firms with higher fixed-cost shares are likely to invest more in research and development. The coefficients are reasonably large: average firms in Compustat invest 3.7% of their sales on R&D over the sample, and this number increases by 0.34 percentage points if the fraction of fixed in total costs increase by 10 percentage points.

Table A6: Relationship between Research & Development and Ratio of Fixed- to Total Costs

	I	II	III	IV
French Firms in FICUS-FARE (1996-2016)				
Fixed-Cost Share	0.024***	0.023***	0.027***	0.019**
	(0.001)	(0.001)	(0.005)	(0.005)
R^2	0.007	0.012	0.003	0.016
Observations	92,536	92,536	92,536	92,536
U.S. Compustat Firms (1980-2016)				
Fixed-Cost Share	0.114***	0.106***	0.037***	0.034***
	(0.003)	(0.003)	(0.003)	(0.003)
R^2	0.16	0.17	0.13	0.15
Observations	125,231	125,231	125,231	125,231
Year fixed effects	No	Yes	No	Yes
Firm fixed effects	No	No	Yes	Yes
Size polynomial	Yes	Yes	Yes	Yes

Notes: Firm-clustered standard errors in parentheses. ** and *** denote significance at the 5 and 1% level, respectively. Size is controlled for through a third degree polynomial in log real sales. Variables are winsorized at 1% and 99% tails.

[48] According to U.S. accounting standards research and development is expensed in Compustat and therefore negatively affects profit. I correct for this by adding *xrd* to the profitability measure in equation (3).

Table A7 presents the regression coefficients from an estimation of equation (7) with the growth of sales as an alternative dependent variable. The explanatory variable is lagged to prevent a mechanically negative relationship through sales shocks, because fixed costs as a percentage of total costs fall inherently when sales increase unexpectedly.[49] Though point estimates vary, there is a clear positive relationship between growth and fixed costs. Jointly, the correlations in this appendix support the mechanisms on which the model relies.

Table A7: Relationship between Sales Growth and Ratio of Fixed- to Total Costs

	I	II	III	IV
French Firms in FICUS-FARE (1994-2016)				
Lagged Fixed-Cost Share	0.155***	0.155***	0.455***	0.514***
	(0.001)	(0.002)	(0.002)	(0.002)
R^2	0.082	0.084	0.057	0.049
Observations	8,670,007	8,670,007	8,670,007	8,670,007
U.S. Compustat Firms (1980-2016)				
Lagged Fixed-Cost Share	0.125***	0.132***	0.055***	0.107***
	(0.009)	(0.009)	(0.025)	(0.025)
R^2	0.014	0.037	0.13	0.15
Observations	111,397	111,397	111,397	111,397
Year fixed effects	No	Yes	No	Yes
Firm fixed effects	No	No	Yes	Yes
Size polynomial	Yes	Yes	Yes	Yes

Notes: Firm-clustered standard errors in parentheses. *** denotes significance at the 1% level. Size is controlled for through a third-degree polynomial in log real sales. Variables are winsorized at 1% and 99% tails.

[49] A lag is furthermore appropriate because the effect of higher R&D investments by high-ϕ firms is unlikely to be immediate. Taking additional lags (e.g. the second or third) rather than the first lags also yields a significantly positive relationship between fixed costs and sales growth.

UniCredit Foundation

Piazza Gae Aulenti, 3

UniCredit Tower A

20154 Milan

Italy

Giannantonio De Roni – *Secretary General*

e-mail: giannantonio.deroni@unicredit.eu

Annalisa Aleati - *Scientific Director*

e-mail: annalisa.aleati@unicredit.eu

Info at:

www.unicreditfoundation.org

UniCredit Foundation