UniCredit Foundation

"Uncertain Product Availability in Search Markets"

Atabek Atayev
Winner 6th Belgrade Young Economist Prize

Working Paper Series

n. 117 ■Aprile 2019

Statement of Purpose

The Working Paper series of the UniCredit Foundation is designed to disseminate and to provide a platform for discussion of either work of the UniCredit economists and researchers or outside contributors (such as the UniCredit Foundation scholars and fellows) on topics which are of special interest to the UniCredit Group. To ensure the high quality of their content, the contributions are subjected to an international refereeing process conducted by the Scientific Committee members of the Foundation.

The opinions are strictly those of the authors and do in no way commit the Foundation and UniCredit Group.

These Working Papers often represent preliminary work. Citation and use of such a paper should take account of its provisional character.

The Working Papers are also available on our website (http://www.unicreditfoundation.org)

Uncertain Product Availability in Search Markets *

Atabek Atayev †

Vienna Graduate School of Economics (VGSE), University of Vienna, Austria

September 22, 2020

Abstract

In many markets buyers are poorly informed about which firms sell the product (product availability) and prices, and therefore have to spend time to obtain this information. In contrast, sellers typically have a better idea about which rivals offer the product. Information asymmetry between buyers and sellers on product availability, rather than just prices, has not been scrutinized in the literature on consumer search. We propose a theoretical model that incorporates this kind of information asymmetry into a simultaneous search model. Our key finding is that greater product availability may harm buyers by mitigating their willingness to search and, thus, softening competition.

JEL Classification: D43, D82, D83

Keywords: Consumer Search; Uncertain Product Availability; Information Asymmetry.

* I thank seminar participants at the Vienna Graduate School of Economics and Kelley School of Business for their helpful comments, especially Jackson Dorsey, Daniel Garcia, Marc Goni, Rick Harbaugh, Maarten Janssen, Aaron Kolb, Eeva Mauring, Marilyn Pease, Eric Rasmusen, Karl Schlag, Matan Tsur, Stephanos Vlachos, and Matthijs Wildenbeest. I am also grateful for helpful discussions by participants of QED/Jamboree 2019, Belgrade Young Economists Conference 2019, EEA-ESEM 2019, EARIE 2019, 14th BiGSEM Doctoral Workshop on Economic Theory, and especially Alistair Macaulay, Anna Obizhaeva, David Ronayne, Andrew Rhodes, and Chris Wilson. Financial support from uni:docs Fellowship Program and research project "Information acquisition, diffusion, and provision" from the University of Vienna is acknowledged.

†Corresponding author. E-mail: atabek.ataev@gmail.com

1 Introduction

Consider a consumer who wishes to renovate a room in her apartment. Typically, the consumer contacts several companies that may potentially provide the service. The companies pass by and then decide whether or not to submit a price quote. Contacting firms and having them come for a visit involves a significant time cost to the consumer. The main reasons why she sends several requests are that, first, she wants to make sure that at least one of them supplies the service and, second, she wants to have competitive bids. Some of the companies that are contacted may not submit a price quote, as they either do not provide the service or may be too busy with other projects. Uncertain product availability refers to the fact that consumers do not know whether or not a firm can compete for the service they request.

There are two features of this example that we focus on in this paper. First, as it takes time for firms to provide price quotes, consumers typically engage in simultaneous search. Second, renovation companies are likely to be better informed than consumers about which of their rivals can provide a certain type of renovation service and are not constrained at that particular moment. This can be justified, for instance, by the estimate from IBISWorld that the annual spending for acquisition of competitor information by companies in the USA was at $ 2 billion in the first half of 2010s (see, Gilad (2015)). Thus, there is asymmetric information between buyers and sellers about product availability in this market. There are many examples of search markets that share these two features. One important set of markets are procurement markets, where a government agency actively solicits firms to compete for a certain procurement project. The agency may find that some of the firms that are actively solicited may not participate in the auction, because their current engagements do not allow them to get involved in more projects, i.e., they are capacity constrained.

This paper is the first one to examine such markets. Product availability clearly affects both supply- and demand-side behavior. Products may be more available for a variety of reasons: there may be a reduction in market demand due to an outflow of residents, an improvement in matching technology makes it easier for consumers to find the desired product, a technological shock that makes production and logistics more efficient may reduce long-lasting capacity constraints, or more firms may have entered the market for the product because of lowered costs of obtaining a license. If products are generally more available, then consumers may be inclined to solicit fewer firms to make price quotes, as it is more likely that a given firm will submit a price quote. On the other hand, whether or not to solicit more price quotes also depends on the chance of getting even better prices. Firms may be inclined to compete more intensely when it is more likely that their competitors will also be able to provide the services, which may reduce price dispersion and the chance of getting better price quotes. This in turn affects the search behavior of consumers.

Following Stigler (1961), MacMinn (1980), Burdett and Judd (1983) and others, we propose a model where consumers search simultaneously and where there is asymmetric information regarding product availability. In the model, a given firm either supplies the product, or does not sell it. All firms know which of them supply the product. We let N represent the total number of firms and θ_n stand for the probability that n out of N firms supply the product. Thus we say that a probability distribution that assigns a greater probability weight to a higher number of sellers supplying the product, i.e., a higher n, makes the product more available to consumers. Ex-ante, consumers are uninformed about who sells the product and what their prices are. In order to make a purchase, a consumer has to observe the price of at least one seller. Consumers can search firms in a non-sequential manner, i.e., they choose a number of firms to search prior to receiving any response from firms. Consumers incur the search cost independent of whether the searched firm has or does not have the product.

We show that, for sufficiently small search costs, there exists a symmetric equilibrium where some consumers actively search more than one firm. Search strategy of consumers in such equilibrium has a specific feature. In equilibrium, either all consumers search the same number of firms, or they randomize over searching two different numbers of firms.

Our key finding is that greater product availability may raise prices. In particular, if a higher product availability means that θ_n decreases for small n where $n \geq 2$ and increases for high n, the expected price increases. For instance, if it is less likely that duopoly sellers supply the product and more likely that triopoly sellers do, sellers' overall market power rises for sufficiently small search costs. There are two effects. First, there is a direct competition effect. As, keeping all else equal, the expected price declines with the number of sellers, consumers are more likely to pay lower prices because they are more likely to face a market with a larger number of sellers. There is, however, also an indirect search effect, which is anti-competitive. Oligopoly markets with more sellers and lower prices are also characterized by lower levels of price dispersion. Thus, consumers have less incentive to search and search intensity drops as the expected incremental gain of an additional search is relatively small. This search effect implies that consumers compare fewer prices, increasing the market power of sellers. We show that this (indirect) anti-competitive effect of greater product availability dominates the (direct) competitive effect if the search cost is relatively small.

The result has implications for procurement markets and other markets where these two features apply. Often, these markets are characterized by low search costs relative to the value of the product. In markets for renovation services, search costs can be associated with finding potential providers' emails online and sending them messages.

In procurement markets, the value of the project for which procurement is being made is usually much higher than the cost of soliciting bidders. Moreover, in both types of markets, greater product availability typically means that a greater number of oligopoly sellers are likely to offer the product.

The intuition behind the detrimental impact of a higher product availability on competition is similar to those in studies by Fershtman and Fishman (1994) and Armstrong et al. (2009). These papers report that price caps raise the expected price in non-sequential search markets. The reason is that effective price caps guarantee low prices and, thus, reduce price dispersion, which has a detrimental effect on the search incentives of consumers. Just as price caps alleviate price dispersion, an increase in the probability of more sellers carrying the product on their shelves makes prices less dispersed in our model.

Our main result is robust assumptions of the model. Particularly, we allow for search cost heterogeneity by introducing a share of consumers whose search cost is set to zero. We report that greater product availability harms consumers with positive search costs. Furthermore, we consider markets where consumers employ *newspaper* search. According to this search protocol, each consumer receives an information about product availability and a price, if the product is available, of a random firm and then decides whether to access a price aggregator to learn product availability and prices of all the other firms. We demonstrate that greater product availability makes consumers worse-off.

We discuss our paper's contribution to the literature in the following section. In Section 3 we present the model. We provide an equilibrium analysis in Section 4 and present comparative statics results in Section 5. In Sections 6 and 7, we provide robustness checks for different model extensions. The final section concludes.

2 Related Literature

Our paper contributes to several strands of the literature. One is the consumer search literature with uncertain product availability and, within this field, the studies by Janssen and Non (2009) and Lester (2011) are the closest to our paper. The main difference between these papers and ours is that they do not consider asymmetric information: an individual seller, just like buyers, does not observe which other sellers offer the product. Therefore, sellers cannot condition their prices on the total number of sellers in the market. For instance, if there happens to be a single seller in the market, the monopolist simply does not know this fact and, in equilibrium, does not set the monopoly price. There are also other important differences. Specifically, Janssen and Non (2009) restrict their attention to two potential sellers, which makes the impact of greater product availability on competition straightforward. Precisely, the more available the product (or the more probable that there are two sellers rather than a monopolist), the stronger the competition.

Lester (2011) examines a model with exogenously given shares of consumers who observe a single price and those who compare prices in markets with firms which have limited capacity. The author reports that an (exogenous) increase in the share of consumers who compare all prices does not necessarily lead to more competition. Other papers which study uncertain product availability in consumer search markets, but without the information symmetry, include Janssen and Rasmusen (2002), Rhodes (2011), and Gomis-Porqueras et al. (2017).

There is a large body of literature that studies search frictions and uncertainty about product availability in labor markets. In these studies, uncertainty is related to availability of a vacant job position. However, these studies do not consider information asymmetry on job availability. We refer to Wright et al. (2017) for an excellent review of the literature.

There is another strand in the consumer search literature studying information asymmetries between buyers and sellers. Yet these studies focus on information asymmetry on either marginal costs of production (e.g., Benabou and Gertner (1993), Dana (1994), Tappata (2009), and Janssen et al. (2011)), or product quality (e.g., Hey and McKenna (1981), Pesendorfer and Wolinsky (2003), Wolinsky (2005), Fishman and Levy (2015)).

In a broader sense, associating entry of firms with a greater product availability, the paper also contributes to the literature that studies the effect of entry on competition (e.g., Janssen and Moraga-Gonzalez (2004), Chen and Riordan (2008), Gabaix et al. (2016), Moraga-Gonzalez et al. (2017), Chen and Zhang (2018)). The papers closest to ours are ones by Janssen and Moraga-Gonzalez (2004) and Moraga-Gonzalez et al. (2017). The essential difference between these papers and ours is that, in these papers, both consumers and sellers know which sellers offer the product. These studies extend the traditional model of non-sequential search by introducing search cost heterogeneity. Janssen and Moraga-Gonzalez (2004) let a share of consumers have zero search costs and report that, if search costs are small, the expected price is non-monotonic with respect to the number of sellers. Moraga-Gonzalez et al. (2017) assume that a consumer's search cost is a draw from some distribution and demonstrate that, if consumers have similar search costs, an additional firm entry results in lower prices. Unlike Janssen and Moraga-Gonzalez (2004), we show that the expected price rises and buyers are worse-off under sufficiently small search costs. In contrast to Moraga-Gonzalez et al. (2017), we find that the expected price can rise in markets with homogeneous, yet sufficiently small, search costs.

The only other paper that accounts for information asymmetry on product availability in search markets is Parakhonyak and Sobolev (2015). The authors also assume that buyers do not know how many sellers there are, but where we analyze this question in a more traditional Bayesian game of incomplete information, they assume that buyers (who search sequentially) do not have a prior and want to minimize *regret* instead of maximizing utility. The authors report that the equilibrium expected price paid by buyers is invariant to changes in product availability.

3 Model

In our model, there are $N \geq 3$ potential sellers, which we call *firms*. N is assumed to be finite. Nature chooses n number of entrants, or simply *sellers*, where $0 \leq n \leq N$. The probability with which nature chooses n entrants is given by θ_n, so that $\sum_{n=0}^{N} \theta_n = 1$. Let $\theta \equiv (\theta_0, \theta_1, ..., \theta_N)$ represent a vector of such θ_n. Each firm observes who has entered the market. Sellers produce homogeneous goods at a marginal cost normalized to zero and compete on prices to maximize profits. Since mixed strategies are allowed, let $x_{nj}(p)$ be the probability that seller j charges a price greater than p when there are n number of sellers in the market.

The demand side of the market is represented by a unit mass of consumers, or *buyers*. Each consumer has an inelastic demand for a unit of a product, which she values at $v > 0$.[1] Ex-ante, consumers do not know which (if any) firms are active sellers as well as what the sellers' prices are in the market. In order to buy a product, a consumer has to engage in costly search and learn at least one price. A search is of fixed-sample-size, where consumers commit to visit (search) k number of firms. We let $c > 0$ denote the search cost. Following the majority of literature on consumer search, we assume that searching one firm is free.[2] Finally, let q_{ki} stand for the probability that consumer i searches k firms so that q_i represents the search probability distribution.

It is useful to note that the probability that a consumer observes m prices, when searching k firms in a market with n sellers, follows a hypergeometric series. This probability, denoted by $\alpha_{nk,m}$, is

$$\alpha_{nk,m} = \frac{\binom{N-n}{k-m}\binom{n}{m}}{\binom{N}{k}}.$$

Then, for any two positive integers I_1 and I_2, we let $\binom{I_1}{I_2} = 0$ for $I_1 < I_2$ and define

$$\alpha_{nk}(x) \equiv \sum_{m=0}^{n} \alpha_{nk,m} x(p)^m$$

[1] We can think of v as the effective reservation price of consumers. Specifically, if we let r be the actual reservation price and $w > 0$ be the outside option of each consumer, then $v = r - w$. Clearly, for $w \geq r$, buyers never participate in the market. The paper focuses on the interesting case of $r > w$.

[2] This assumption does not affect the main results qualitatively. If we allow each search to be costly, then in the trivial equilibrium stated in Proposition 1 buyers do not search and, thus, there is no trade.

to be the probability generating function, where $x(p)^m \equiv [x(p)]^m$.[3]

The timing of the game is as follows. First, nature chooses a number of sellers that enter the market. Each firm observes whether itself as well as any other firms entered the market. Consumer do not have this information. Second, sellers simultaneously set prices. Third, without knowing prices, consumers choose the number of firms to visit. Consumers who observe at least one price may make a purchase. Finally, the payoffs are realized.

We employ symmetric Bayesian-Nash equilibrium (SBNE) as a solution concept. Therefore we drop subscripts j and i to simply write x_n and q, respectively. Let p_{-j} be the vector of prices set by other sellers than seller j. (Clearly, if there is a monopolist seller, p_{-j} is an empty set.) Also let $\Pi_{nj}(p, p_{-j})$ denote the expected profit of seller j that charges p, given pricing strategies of the other sellers, in a market with n sellers. Then, letting $\overline{\Pi}_n \geq 0$ be some constant for each n, we define an SBNE as a collection of price distributions $(x_n)_{n=1}^N$ and search probability distribution q such that for each n (a) $\Pi_{nj}(p, p_{-j}) \geq \overline{\Pi}_n$ for all p in the support of $x_n(p)$, $\forall j$, and (b) $\Pi_{nj}(p, p_{-j}) \leq \overline{\Pi}_n$ for all p, $\forall j$; (c) each consumer searching k firms obtain no lower utility by searching any other number of firms for all $q_k > 0$ and (d) $\sum_{k=0}^N q_k = 1$.

4 Equilibrium Analysis

We start our analysis by identifying consumers' search strategies which can be a part of an SBNE. Subsection 4.1 serves this purpose. There, we first demonstrate an existence of a trivial SBNE where consumers do not search more than one firm. As we are not interested in this equilibrium, we next focus on SBNEs where $q_0 + q_1 < 1$, i.e., consumers search *actively*. We show that in any SBNE with active search, buyers either search k firms where $2 \leq k \leq N-1$ or randomize over searching k and $k+1$ firms where $1 \leq k \leq N-1$.

We then proceed to construct those two types of SBNEs with active search. To do that, we employ the following steps. In Subsection 4.2, we assume that consumers randomize between searching k and $k+1$ firms, and find the optimal pricing strategies of sellers. Given these pricing strategies, we check whether consumers indeed find it optimal to

[3] Also it is easy to see that $\alpha_{nk}(x)$ is closely related to the Gauss hypergeometric function. If

$$ {}_2F_1(a, b; c; x) \equiv \sum_{m=0}^{n} \frac{(a)_m (b)_m}{(c)_m m!} x^m(p) $$

represents the Gauss hypergeometric function, where $(a)_m = a(a+1)...(a+m-1)$, then it follows that

$$ \alpha_{nk}(x) = \frac{\binom{N-n}{k}}{\binom{N}{k}} {}_2F_1(-n, -k; N-n-k+1; x). $$

randomize over searching k and $k+1$ firms. In Subsection 4.3, we apply the same steps, yet we consider the case where all buyers search k firms.

We demonstrate that an SBNE with active search definitely exists if the search cost is not too high. Generically, there is multiplicity of equilibria. We establish that a locally stable SBNE with active search is unique for sufficiently small search costs.

4.1 Preliminary Results

Our first result is that there always exists an equilibrium where consumers search at most one firm. That buyers do not search more than one firm means that they do not compare any prices. Then, it is optimal for sellers to charge the monopoly price v. Such pricing clearly justifies the above search strategy of buyers, as buyers receive zero payoff both when they purchase a product and when they do not, yet searching more than one firm is costly. This is a well-known result in models of both sequential (Diamond (1971)) and simultaneous search (Burdett and Judd (1983)).

Proposition 1. *For any $v > 0$, $c > 0$ and θ, there exists an equilibrium where sellers set the monopoly price v and consumers search at most one firm: $q_0 + q_1 = 1$.*

We are interested in SBNEs where consumers search more than one firm. Our next two results limit search strategies of buyers which can be part an SBNE with active search. To state the results, we let $\underline{n} \geq 2$ represent the lowest number of oligopoly sellers that are drawn into the market with a strictly positive probability. This implies that $\theta_2 = ... = \theta_{\underline{n}-1} = 0$ while $\theta_{\underline{n}} > 0$.

Lemma 1. *For any $c > 0$ and $\underline{n} \geq 2$, in an SBNE it cannot be that $\sum_{k=N-\underline{n}+2}^{N} q_k = 1$.*

The reasoning is by contradiction. Assume that $\sum_{k=N-\underline{n}+2}^{N} q_k = 1$. Then, the optimal pricing strategies of sellers are as follows. The monopolist sets v. Sellers in a market with $n \geq \underline{n}$ optimally charge a price equal to the marginal cost of production. The argument stems from the observation that, in these markets, an individual seller's price is always compared with at least one other price (it is easy to check that consumers observe at least two prices). Therefore, an individual seller does not want to be the highest priced one and, in the case of a tie in prices, undercutting is profitable. As a result, the sellers price at the production marginal cost. Given the above pricing strategies of sellers for different n, it is easy to see that a buyer who searches $N - \underline{n} + 2$ receives a payoff equal to $(1 - \theta_0 - \theta_1)v - (N - \underline{n} + 1)c$. If she searches $N - \underline{n} + 1$ firms instead, she receives a payoff equal to $(1 - \theta_0 - \theta_1)v - (N - \underline{n})c$. Clearly, the latter payoff is greater than the former. This is a contradiction to the initial assumption that, in equilibrium, buyers search at least $N - \underline{n} + 2$ firms.

The following proposition narrows down even more the search strategies of buyers in an SBNE, where some consumers search actively.

Proposition 2. *An SBNE with $q_0 + q_1 < 1$ can exist if, and only if, $\theta_0 + \theta_1 < 1$. In any such SBNE,*

(i) it must be that either x_n is continuous and strictly decreasing in p and satisfies $x_n(v) = 0$, or x_n must have a unit mass at either $p = 0$ or $p = v$;

(ii) there exists k such that for $2 \leq k \leq N - \underline{n} + 1$ it must be that $q_k = 1$, or for $1 \leq k \leq N - \underline{n} + 1$ it must be that $0 < q_k < 1$ and $q_k + q_{k+1} = 1$.

If $\theta_0 + \theta_1 = 1$, consumers receive zero payoff independent of how many firms they search. This is because they either find a product and make a purchase at price v or do not find a product. Then, as searching more than one firm is costly, buyers optimally search at most one firm. Thus, an SBNE with active search exists only if $\theta_0 + \theta_1 < 1$.

The reasoning behind (i) can be understood as follows. Clearly, a monopolist seller always charges v. Thus, there is a unit mass at a price equal to v. From Proposition 1 and Lemma 1, it follows that $q_0 + q_1 + \sum_{k=N-\underline{n}+2}^{N} q_k < 1$, i.e., $q_k > 0$ for some $2 \leq k \leq N - \underline{n} + 1$. Let $\widetilde{k}$ be the smallest of such integers such that $q_{\widetilde{k}} > 0$. Then, notice that each seller in a market with n number of sellers, where $\underline{n} \leq n \leq N - \widetilde{k} + 1$, have a (strictly) positive share of consumers observing only its price. These buyers—also known as *locked-in* buyers in the literature—are a source of the sellers' market power. Each of these sellers also face buyers who observe at least one other price in addition to the seller's price. As sellers which set higher prices are less likely to sell to the price-comparing consumers, these buyers put competitive pressure on the sellers. The existence of locked-in and price-comparing consumers gives rise to price dispersion. Finally, if $q_0 = q_1 = ... = q_{\widetilde{k}-1} = 0$ and $q_{\widetilde{k}} > 0$, then consumers compare at least two prices in a market with n number of sellers where $n \geq N - \widetilde{k} + 2$. Hence, for $n \geq N - \widetilde{k} + 2$, sellers must charge a price equal to the production marginal cost in equilibrium as we argued in the paragraph after Lemma 1. Thus, in these markets there is a unit probability mass at a price equal to 0.

In markets with equilibrium price dispersion (and n sellers), an individual seller must be indifferent of setting any price in the support of price distribution x_n. Then, x_n must be atomless because if it had an atom, undercutting would be beneficial due to the strictly positive share of consumers who compare at least two prices. Also x_n cannot have a flat region in the support, else an individual seller will not be indifferent at both ends of that flat region. Furthermore, the highest price in the support of the price distribution must be equal to v. It cannot exceed v, since a seller charging a price higher than v does not sell to anyone. The upper bound cannot be less than v because if it were, a seller could improve its profit by deviating to v, as its expected demand in both cases consists of only locked-in buyers.

Finally, we establish the understanding behind (ii) from the above fact that price distribution is non-degenerate in markets for certain realizations of n. This means that the incremental expected benefit of searching one more firm is declining with the number of searches. Since the incremental cost of searching is constant (and is equal to c), it must be that either all consumers search the same number of firms or a share of consumers search k firms while the rest search $k+1$ firms.

This proposition provides us with a great deal of information about search strategies of consumers in SBNEs with active search but relative little information about conditions under which such SBNEs may exist. The following two subsections address these issues.

4.2 Mixed Search Strategy

We start considering the case where consumers play mixed strategies. Suppose buyers randomize between searching k and $k+1$ firms, where $1 \leq k \leq N - \underline{n} + 1$. What is the optimal pricing strategy of sellers? Obviously, the monopolist seller always charges v. If $n \geq N-k+2$ is realized, consumers compare at least two prices. Then, the sellers have an incentive to undercut prices up to the marginal cost of production, thus the equilibrium price being zero. Finally, when there are n sellers such that $2 \leq n \leq N-k+1$ sellers in the market, they set prices from price distribution x_n. The symmetric equilibrium strategy of sellers is such that an individual seller is indifferent between setting any price in the support of the price distribution and must (weakly) prefer these prices to ones which are not in the support.

To derive x_n for $2 \leq n \leq N-k+1$, we first note that a consumer who searches k firms buys from seller j if she visits the seller and observes no lower price than the seller's price. Therefore, seller j pricing at p sells to this consumer with probability

$$\sum_{m=1}^{n} \frac{\binom{N-n}{k-m}\binom{n}{m}m}{\binom{N}{k}n} x_n(p)^{m-1} = \frac{1}{n}\sum_{m=0}^{n} \frac{\binom{N-n}{k-m}\binom{n}{m}}{\binom{N}{k}} m x_n(p)^{m-1} = \frac{\alpha'_{nk}(x_n(p))}{n}.$$

We next let $\beta_{nk}(x) \equiv q_k\alpha_{nk}(x)+(1-q_k)\alpha_{nk+1}(x)$ so that $\beta_{nk,m} \equiv q_k\alpha_{nk,m}+(1-q_k)\alpha_{nk+1,m}$ is the total share of consumers who observe m prices. Then, seller j that sets price p expects to earn

$$\Pi_{nj}(p, p_{-j}) = p\frac{(\beta_{nk,1} + 2\beta_{nk,2}x_n(p) + 3\beta_{nk,3}x_n(p)^2 + ...)}{n} = \frac{\beta'_{nk}(x_n(p))p}{n}.$$

As an individual seller is indifferent in terms of setting any price in the support of equilibrium distribution function, it follows

$$p\beta'_{nk}(x_n(p)) = v\beta'_{nk}(x_n(v)). \tag{1}$$

This equation implicitly (and uniquely) defines equilibrium $x_n(p)$ (see, Johnen and Ronayne (2020) for uniqueness of x_n). For convenience, we will use the inverse function $p_n(x_n)$, which in equilibrium satisfies

$$p_n(x_n) = \frac{n\overline{\Pi}_n}{\beta'_{nk}(x_n)}.$$

Then, the lower bound of the price distribution, denoted by $\underline{p}_n$, solves $\underline{p}_n = p_n(1)$.

Now, it is left to check whether consumers indeed randomize between searching k and $k+1$ firms if sellers price the product as discussed above. For that, we first note that as the density of the lowest of m prices is

$$\frac{d}{dp}\left(1 - x_n(p)^m\right) = -mx_n(p)^{m-1}x'_n(p),$$

the expected price paid by a buyer searching k firms in a market with n sellers is

$$-\sum_{m=1}^{n}\int_{\underline{p}_n}^{v} p\alpha_{nk,m} m x_n(p)^{m-1}x'_n(p)dp = -\int_{\underline{p}_n}^{v} p\alpha'_{nk}(x_n(p))x'_n(p)dp = \int_0^1 p_n(x_n)\alpha'_{nk}(x_n)dx_n,$$

where we obtained the last equality by changing variables from p to $x_n(p)$. As not visiting a seller is equivalent to paying price v, we define the expected *virtual* price paid by a consumer who searches k firms as

$$P_k \equiv (\theta_0 + \theta_1)v + \sum_{n=\underline{n}}^{N-k+1}\theta_n\left(\alpha_{nk,0}v + \int_0^1 p_n(x_n)\alpha'_{nk}(x_n)dx_n\right).$$

Here, we used the fact that pricing policies of sellers in markets with n sellers such that $1 < n < \underline{n}$ are irrelevant for consumers as, by the definition of $\underline{n}$, we have $\theta_n = 0$ for all $1 < n < \underline{n}$. Next, the expected virtual price paid by consumers who search $k+1$ firms can be expressed as above by changing the respective indices from k to $k+1$. Then, the incremental benefit of searching the $k+1$th firm is

$$\begin{aligned} P_k - P_{k+1} &= \sum_{n=\underline{n}}^{N-k+1}\theta_n\left((\alpha_{nk,0} - \alpha_{nk+1,0})v + \int_0^1 p_n(x_n)\left(\alpha'_{nk}(x_n) - \alpha'_{nk+1}(x_n)\right)dx_n\right) \\ &= -\sum_{n=\underline{n}}^{N-k+1}\theta_n\int_0^1 p'_n(x_n)\left(\alpha_{nk}(x_n) - \alpha_{nk+1}(x_n)\right)dx_n, \end{aligned}$$

where the second line follows from integration by parts and the facts that $\alpha_{nk}(0) = \alpha_{nk,0}$,

$\alpha_{nk+1}(0) = \alpha_{nk+1,0}$, $p_n(0) = v$ and $\alpha_{nk}(1) = \alpha_{nk+1}(1) = 1$. Clearly, in equilibrium, it must be that

$$P_k - P_{k+1} = c. \tag{2}$$

The question that comes forward is, does there exist $q_k \in (0, 1)$ that satisfies (2)? The following proposition answers the question.

Proposition 3. *For any $v > 0$, $\underline{n} \geq 2$, θ_0 and θ_1 such that $0 \leq \theta_0 + \theta_1 < 1$, and k such that $1 \leq k \leq N - \underline{n} + 1$, there exists $(\underline{c}_{k,k+1}, \bar{c}_{k,k+1}) \subset (0, v)$ such that for $c \in (\underline{c}_{k,k+1}, \bar{c}_{k,k+1})$ there exists an SBNE given by $((x_n)_{n=1}^{N}, q)$, where*

$$1 - x_1(p) = \begin{cases} 0, & p < v, \\ 1, & p \geq v, \end{cases}$$

$$1 - x_n(p) = \begin{cases} 0, & p < 0, \\ 1, & p \geq 0, \quad \text{for} \quad N - k + 2 \leq n \leq N, \end{cases}$$

$x_n(p)$ is determined by (1) for $2 \leq n \leq N - k + 1$, and q_k is determined by (2), and $q_{k+1} = 1 - q_k$.

Furthermore, $\underline{c}_{k,k+1} = 0$ for $k = 1$ and $k = N - \underline{n} + 1$.

We develop the proof as follows. We first show in the appendix that $P_k - P_{k+1}$ is positive for $0 < q_k < 1$. The intuition is that consumers who search $k + 1$ firms are more likely to find the product and compare prices than buyers who search k firms. We next demonstrate that the left-hand side of (2) is strictly concave in q_k. This means that there can be at most two values of q_k that satisfy (2). Using the above two properties of $P_k - P_{k+1}$, we establish that there must be two cutoff values of search cost $\underline{c}_{k,k+1}$ and $\bar{c}_{k,k+1}$ such that $\underline{c}_{k,k+1} = \min\{\lim_{q_k \downarrow 0}(P_k - P_{k-1}), \lim_{q_k \uparrow 1}(P_k - P_{k-1})\}$ and $\bar{c}_{k,k+1}$ is maximum possible value of $P_k - P_{k+1}$ for $0 < q_k < 1$.

The first part of the proposition informs us of existence of search cost intervals under which equilibria in consumers' mixed strategies exist, but it does not tell us anything about those intervals. The second part of the proposition partially addresses this issue. Namely, it shows that for sufficiently small search costs, an SBNE with positive search definitely exists. We prove this in the appendix by demonstrating $P_1 - P_2$ and $P_{N-\underline{n}+1} - P_{N-\underline{n}+2}$ converge to zero as $q_1 \uparrow 1$ and $q_{N-\underline{n}+2} \uparrow 1$, respectively.

Figure 1 provides an illustration of a solution to (2). The horizontal axis represents the share of consumers who search $k+1 = 3$ firms, while the vertical axis stands for search cost and prices. The solid curve represents $P_k - P_{k+1}$, while the dashed ones stand for the search costs. Notice that $P_k - P_{k+1}$ is positive and concave in $q_k \in (0, 1)$. It is easy to see that $\underline{c}_{k,k+1} = 0$ in this particular case. This means that for sufficiently small search costs

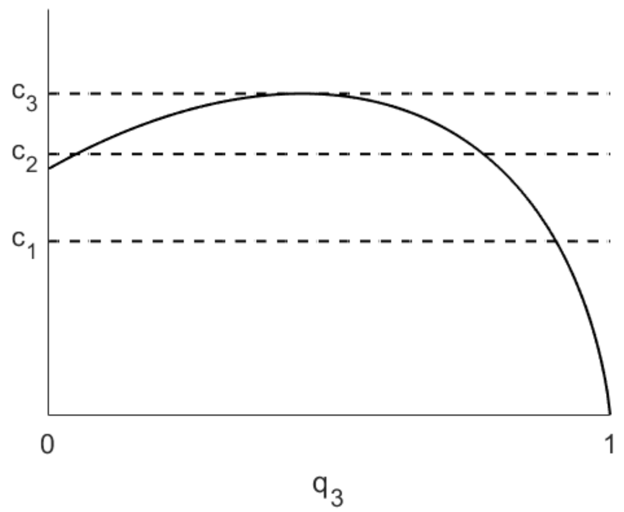

Figure 1: Illustration of SBNEs for $N = 3$, $k = 2$, $v = 1$, $\theta_0 = 0$, $\theta_1 = \theta_3 = 0.05$, and $\theta_2 = 0.90$

an SBNE where a share of buyers search two firms while the rest search three firms exists. The maximum value of the solid curve, which corresponds to $c_3(= 0.055)$, represents $\bar{c}_{k,k+1}$. For the value of the search cost given by $c_1(= 0.03)$, the dashed and solid lines cross only once. The intersection represents an SBNE. For the value of search cost given by $c_2(= 0.045)$, there are two intersections of the dashed and solid lines, representing two SBNEs.

4.3 Pure Search Strategy

Next, we consider the case where all consumers search k firms, where $2 \leq k \leq N - \underline{n} + 1$. It is easy to see that, if all consumers search k firms, in equilibrium the monopolist seller charges v and sellers price at the marginal cost if $n \geq N - k + 2$. For an intermediate number of sellers $2 \leq n \leq N - k + 1$, price dispersion arises and equilibrium price distribution (in market with n sellers) is determined by

$$p\frac{\alpha'_{nk}(x_n(p))}{n} = v\frac{\alpha_{nk,1}}{n} = \overline{\Pi}_n > 0, \tag{3}$$

where the inequality is due to $\alpha_{nk,1} > 0$.

To check whether buyers indeed visit k firms, given the above pricing strategies of sellers, it suffices to find conditions (if there are such) under which the following set of inequalities hold for $q_k = 1$:

$$\begin{aligned} P_k - P_{k+1} &\leq c, \\ P_{k-1} - P_k &\geq c. \end{aligned} \tag{4}$$

The following proposition shows that there exists a nonempty interval of search costs such that the set of inequalities are satisfied.

Proposition 4. *For any $v > 0$, $\underline{n} \geq 2$, θ_0 and θ_1 such that $0 \leq \theta_0 + \theta_1 < 1$, and k such that $2 \leq k \leq N - \underline{n} + 1$, there exists $[\underline{c}_k, \overline{c}_k] \subset (0, v)$ such that for $c \in [\underline{c}_k, \overline{c}_k]$ there exists an SBNE given by $\left((x_n)_{n=1}^N, q\right)$ where*

$$1 - x_1(p) = \begin{cases} 0, & p < v, \\ 1, & p \geq v, \end{cases}$$

$$1 - x_n(p) = \begin{cases} 0, & p < 0, \\ 1, & p \geq 0, \quad \textit{for} \quad N - k + 2 \leq n \leq N, \end{cases}$$

$x_n(p)$ is determined by (3) for $2 \leq n \leq N - k + 1$ and $q_k = 1$.

It is crucial to point out the relationship of cutoff search costs $\underline{c}_k$ and $\overline{c}_k$ to those in the previous subsection $\underline{c}_{k,k+1}$ and $\overline{c}_{k,k+1}$. Observe that the search cost under which consumers randomize between searching k and $k+1$ firms with $q_k \uparrow 1$ in equilibrium is equal to $\underline{c}_k$. Similarly, the search cost under which buyers randomize as above but with $q_k \downarrow 0$ in equilibrium is equal to $\overline{c}_{k+1}$. These two observations mean that it is impossible that, for certain regions of search costs, there is no equilibrium where consumers randomize between choosing k and $k+1$ or choose k only, but there are equilibria for values of search costs which are above and below that region: one characterized by buyers randomizing over k and $k+1$ and the other by buyers choosing k. Then, recalling the second part of Proposition 3, we can conclude that an equilibrium with active search exists if search cost is not very high.

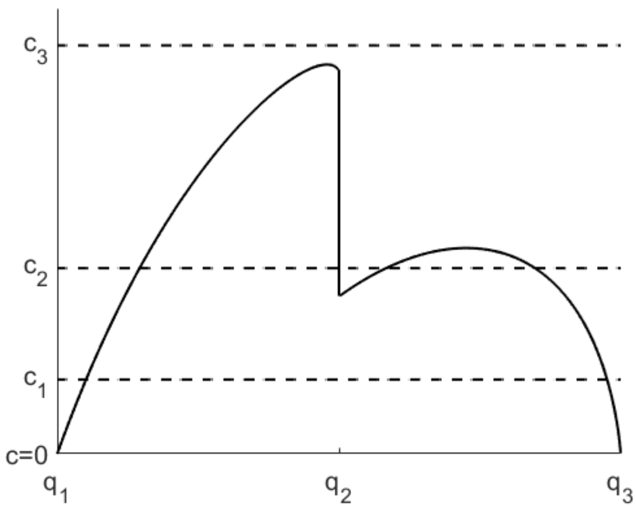

Figure 2: Illustration of a multiplicity of SBNE for different search costs.

Figure 2 graphically illustrates the main idea of the discussion. (We used the same parameter values as in Figure 1, with exceptions that we set $c_1 = 0.02$, $c_2 = 0.05$, and $c_3 = 0.11$.) The horizontal axis represents q_n for $n \in \{1, 2, 3\}$. At each of the three points on the axis, i.e., at each q_n, we have $q_n = 1$. When we move to the left or right of that point along the axis, q_n starts decreasing and q_{n-1} or q_{n+1}, respectively, starts increasing. For example, start with point q_2 on the horizontal axis, which means that $q_2 = 1$ and $q_1 = q_3 = 0$. If we gradually move to the left along the axis, q_2 begins decreasing while q_1 begins increasing so that $q_1 + q_2 = 1$ and $q_3 = 0$. The vertical axis of the graph represents the search cost and the expected benefit of searching one more firm. The solid curve represents the incremental benefit of searching one additional firm. Observe that any point on the solid line that corresponds to a point between q_k and q_{k+1} on the horizontal axis represents the incremental benefit of searching the $k + 1$th firm. The dashed lines stand for different levels of search cost. Each intersection of solid curve and a dashed line represents an equilibrium for that particular value of search cost. Importantly, we note the following two points. The solid line is continuous over q_ns with $\lim_{q_2 \uparrow 1}(P_1 - P_2) = \bar{c}_2$ and $\lim_{q_2 \uparrow 1}(P_2 - P_3) = \underline{c}_2$. Moreover, we have $\lim_{q_1 \uparrow 1}(P_1 - P_2) = \underline{c}_{1,2} = 0$ and $\lim_{q_3 \uparrow 1}(P_2 - P_3) = \underline{c}_{2,3} = 0$. These points imply that an equilibrium with active search exists if the search cost is not too high, e.g., if $c = c_1$ or $c = c_2$. For instance, for a value of search cost equal to c_3, there is no SBNE with active search.

Subsections 4.2 and 4.3 imply that there is multiplicity of equilibria with active search. This can be easily seen from Figure 2. For instance for a value of search cost equal to c_2, there are four SBNEs with active search. In the following corollary, we state conditions under which there is a unique stable SBNE with active search. The corollary is implied by propositions 3 and 4.

Corollary 1. *For $v > 0$, $\underline{n} \geq 2$, θ_0 and θ_1 such that $\theta_0 + \theta_1 < 1$ and $c \in (0, \min\{\underline{c}_2, \underline{c}_{N-\underline{n}+1}\})$, there exists a unique locally stable SBNE where consumers randomize over searching $N - \underline{n} + 1$ and $N - \underline{n} + 2$ firms.*

We again refer to Figure 2 to illustrate the intuition behind the proposition. We first observe that for sufficiently small search costs, e.g., c_1, there are two equilibria: the leftmost one being unstable, and the rightmost one being stable. Consider the rightmost equilibrium. It is easy to see that if the actual probability that consumers search three firms is higher (lower) than the equilibrium one, the incremental benefit of searching the third firm is lower (higher) than the search cost. As a result, consumers have an incentive for less (more) search so that the actual search probability of searching three firm converges to the equilibrium one. By a similar argument, it is easy to see why the leftmost SBNE is not stable. We also observe that, for a value of search cost equal to c_2, there are two stable SBNEs: the rightmost one and the one represented by an intersection of a vertical part of the solid line and the dashed line. Hence, the figure demonstrates that a unique stable SBNE with active search exists only for sufficiently small search costs.

5 Comparative Statics

In this section, we examine an impact of changes in our two exogenous parameters on equilibrium outcomes: θ representing product availability and c representing the search cost. We only consider stable SBNEs. A change in θ can be a product of government policies aimed at easing bureaucratic processes related to market entry. It may also be a consequence of technological advancement or a reduction in industry demand as discussed in the Introduction. In all these cases, the product is likely to become more available to consumers. The intuition tells us that greater product availability should benefit buyers, as they are less likely to exit the market without purchase and, importantly, a greater number of sellers is usually associated with more intense competition. We demonstrate that this does not necessarily have to be the case. Precisely we show that greater product availability may not have any impact on market outcome or even harm buyers. The last result is due to detrimental effect of greater product availability on consumers' willingness to search.

Changes in c may represent technological development, such as shops creating their websites so that consumers can find out whether a shop has a product with the help of several clicks instead of visiting a brick-and-mortar store. Intuitively, and this turns out to be the case, a smaller search cost strengthens buyers' willingness to search, which increases consumers' chances to find the product and compare prices, thus triggering competition.

Following the order of our analysis in Section 4, we first undertake the comparative static analysis in SBNEs where buyers use a mixed strategy. In Subsection 5.2, we examine markets where consumers play a pure strategy.

5.1 Consumers Playing a Mixed Strategy

We focus on an SBNE that results when the search costs are sufficiently small. There are two reasons for this. One is that, following Corollary 1, we can see that there is a unique locally stable SBNE with active search for small search costs. The other reason is that markets that have been mentioned in the introduction are generally characterized by low search costs relative to the value of the product.

We notice that, as $\sum_{n=0}^{N} \theta_n = 1$, an increase in θ_i must be accompanied by a decrease in at least one θ_j, $i \neq j$. In other words, there are numerous ways of considering a change in a single θ_i. To understand the main mechanism through which a change in θ_i affects market outcomes, it is sufficient to focus on a change in θ_i that is associated with an opposite change in only a single θ_j. In this case, it only matters whether $2 \leq i, j \leq N - k + 1$ or not, for $i \neq j$. Then, we need to consider only three cases: neither i nor j is in the set of integers in interval $[2, N - k + 1]$, only i or j is in that set, and both i and j are in that set. We assume that $i > j$ such that an increase in θ_i with the associated equal decrease in θ_j implies that a product is more available.

In the following proposition, we state the main result of the section. Specifically, we identify sufficient conditions under which a greater product availability has a detrimental effect on buyers' well-being.

Proposition 5. *In a stable SBNE where consumers randomize between searching* $N-\underline{n}+1$ *and* $N - \underline{n} + 2$ *firms for* $2 \leq \underline{n} \leq N - 1$*, an increase in* θ_i *with a corresponding decrease in* θ_j *where* $i > j$

(i) decreases the expected price and improves consumers' well-being for $j = 1$*,*

(ii) does not affect the expected price and consumers' well-being for $j \geq \underline{n} + 1$*,*

(iii) raises the expected price and harms consumers' well-being for $j = \underline{n}$*.*

The reasoning behind (i) is straightforward. Note that, in equilibrium, there is price dispersion only in a market with $\underline{n}$ sellers and the price in a market with at least $\underline{n} + 1$ sellers is equal to the production marginal cost. As the expected prices in markets for any realization of $n \geq 2$ is lower than the monopoly price, the direct effect of a decrease in θ_1 is a fall in the expected price. If θ_1 decreases at the expense of $\theta_{\underline{n}}$, there is an additional indirect effect on the expected price. Following the change in θ, consumers search more intensely as they are more likely to face a market with price dispersion. Therefore, sellers in a market with $\underline{n}$ sellers lose their market power because the share of consumers who compare prices rises. Despite the fact that more searching also leads to more resources spent on search, consumers are better off.

The understanding behind (ii) is intuitive. Since the equilibrium price in a market with at least $\underline{n}+1$ sellers is equal to the production marginal cost, a decrease in $j \geq \underline{n}+1$ accompanied by an increase in $i(> j)$ does not change the expected price. Also, it does not affect consumer search behavior, as this change in θ does not affect the expected level of price dispersion. Then, market power of sellers for any realization of n does not change, nor does consumers' well-being.

Finally, the intuition behind (iii) is as follows. First, there is a direct effect on the expected price. Since prices in $j = \underline{n}$ are bounded above the production marginal cost and the equilibrium price in a market with i sellers is equal to zero, the expected price decreases. There is also an indirect effect. Consumers search less following a decrease in θ_j. This is due to the decrease in the likelihood that consumers face a market with equilibrium price dispersion. Less search, on the one hand, raises the market power of sellers in a market with $\underline{n}$ sellers. On the other hand, consumers economize on search costs. The proof, however, demonstrates that the negative effect of greater product availability on buyers' well-being dominates its positive effect.

To illustrate the idea, consider an example with $N = 3$, $c = 0.05v$, and $\theta_2 + \theta_3 = 1$, where θ_3 increases from 0.1 to 0.2. In a unique stable equilibrium with active search, buyers randomize between searching 2 and 3 firms. This means that the prices are dispersed in a duopoly market and triopoly sellers charge a price equal to the marginal cost of production. Following the increase in θ_3, the share of consumers who search all three firms drops from approximately 0.78% to 0.70%, which is around an 11% decrease. The expected price increases by around 14%—from $0.195v$ to $0.222v$. As a result, consumers' surplus (incorporating search costs) falls from approximately $0.716v$ to $0.693v$, which is a decrease of 3.2%.

So far we have considered the impact of marginal changes in θ on market outcomes. Thus, as the final point of the section, we provide some insights on substantial (as opposed to marginal) changes in the product availability on the market outcomes. From Section 4, it follows that there may be two stable SBNE for certain parameter regions. One of the SBNE occurs in pure strategies of consumers, whereas the other in consumers' mixed strategies. With the help of numerical simulations, we report how market outcomes change in those two equilibria.

Figures 3 and 4 illustrate the impacts of greater product availability on the expected price paid by buyers and their well-being. We used the following parameter constellations: $N = 3$, $\theta_0 + \theta_1 = 0$, $v = 1$ and $c = 0.04$. The horizontal axes in the figures represent the value of θ_3. We fix $\theta_2 + \theta_3 = 1$ and increase the value of θ_3 from 0 to around 0.68. The vertical axis in Figure 3 stands for the expected price paid by a random buyer, while in Figure 4 it stands for buyers' surplus. The solid lines represent the respective variables in an SBNE where buyers play mixed strategies. The dashed lines correspond to the respective variables in a SBNE characterized by pure strategies of buyers.

From both figures, we can see that for sufficiently small values of θ_3, there only exists a stable SBNE in mixed strategies of consumers: some buyers searching 2 firms, while the remaining ones 3 firms. In contrast, for moderately high values of θ_3, only a stable SBNE in pure strategies of buyers exists: all buyers search 2 firms. Finally, for moderate values of θ_3, both stable SBNEs exist. In Figure 3, the expected price rises with θ_3 in the SBNE where consumers play a mixed strategy, whereas the expected price falls with θ_3 in the other SBNE. Notice that for the value of θ_3 where both SBNEs exist, the expected price is higher in the equilibrium characterized by pure strategy of buyers than in the other equilibrium. Figure 4 depicts a similar picture as Figure 3. Importantly, consumers' surplus is decreasing with a greater product availability in the SBNE where buyers play

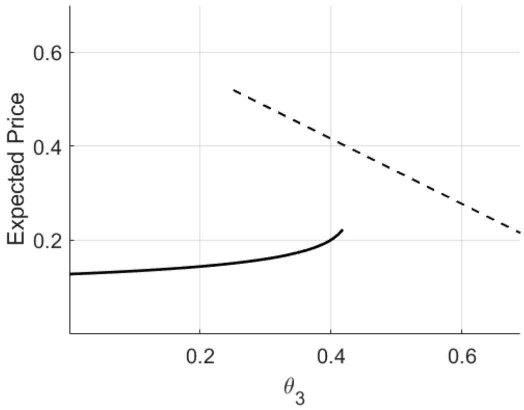

Figure 3: Impact of greater product availability on the expected prices.

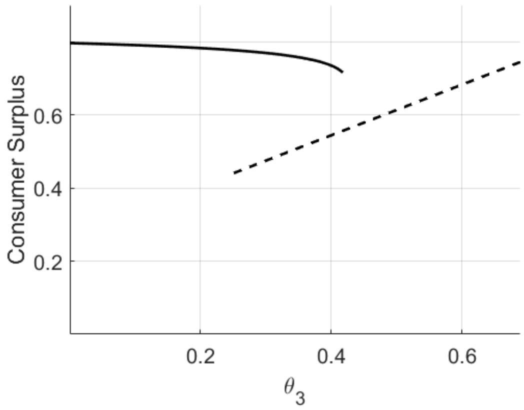

Figure 4: Impact of greater product availability on the buyers' surplus.

mixed strategies, while it is increasing in the SBNE with buyers' pure strategies. For values of θ_3 where both types of equilibria exist, buyers are better off in the SBNE with mixed strategies. This is not surprising, as in the SBNE with consumers' mixed strategies they search more and impose more competitive pressure on sellers than in the SBNE with pure strategies.

Now, we discuss the impact of a change in c on market outcomes.

Proposition 6. *In any stable SBNE characterized by consumers randomizing over searching k and $k+1$ firms, an increase in c causes less search and impairs consumers' well-being.*

It is fairly straightforward that an increase in search cost mitigates consumers' willingness to search. We know that less search is associated with a greater market power of sellers. Also consumers are less likely to find the product. Still consumers spend less resources on search costs. In the appendix, we show that the former two negative effects of an increase in search cost on consumers' well-being dominate the latter positive effect.

5.2 Consumers Playing a Pure Strategy

We continue our comparative static analysis to SBNEs where consumers play a pure strategy. Notice that there is a continuum of search costs under which such an equilibrium exists. This means that marginal changes in θ or c do not affect search behavior of buyers and, therefore, pricing strategies of sellers. Nevertheless, these changes affect consumers' well-being as well as the total expected price paid by consumers, as we show in the following proposition.

Proposition 7. *In any stable SBNE characterized by all consumers searching k firms,*

(i) an increase in θ_i with a corresponding decrease in θ_j does not affect consumers' search behavior and

(a) pushes down the average expected price paid by consumers who make a purchase and improves buyers' well-being for $j \leq N - k + 1$,

(b) has no impact on the expected price paid by consumers who make a purchase and on buyers' well-being for $j \geq N - k + 2$;

(ii) an increase in c does not affect consumers' search behavior, the expected price paid by buyers who make a purchase, and impairs their well-being.

The intuition behind (i) is as follows. The expected price paid by a buyer, conditional on observing at least one price, is

$$\frac{\alpha_{nk,1}}{1 - \alpha_{nk,0}} v.$$

The higher the fraction $\alpha_{nk,1}/(1 - \alpha_{nk,0})$, the greater the expected price paid. The proof shows that this share is decreasing in n, for $1 \leq n \leq N - k + 1$. To illustrate the point, Figure 5 depicts this share (the vertical axis) for different values of n (the horizontal axis). As sellers' pricing strategy and buyers' search strategy remain the same following changes in θ, the more product availability, as in (a), translates into a lower share of buyers who drop out of the market. This, along with the lower expected price conditional on making a purchase, implies that buyers' well-being rises. However, this is not true if a greater product availability is as in (b). In that case, all buyers in both markets, one with i number of sellers and the other with j number of sellers, make a purchase and pay price equal to the marginal cost of production. Hence, the changes in θ_i and θ_j do not affect market outcomes. In Figure 5, this is illustrated by $j = 9$ and $i = 10$.

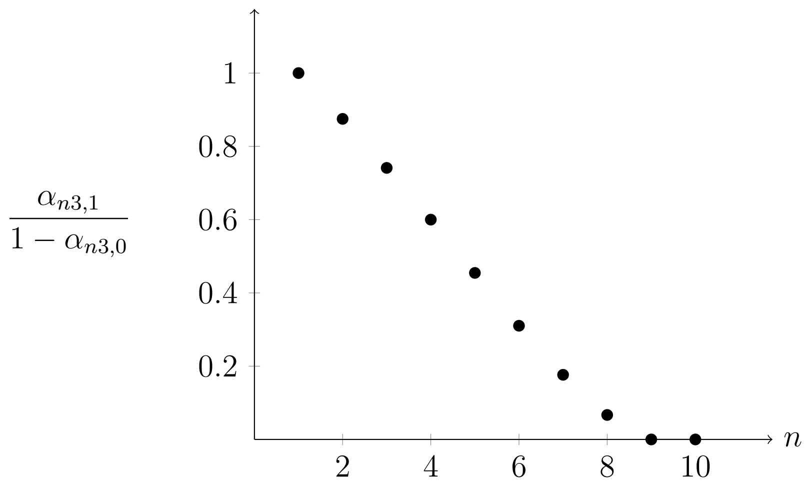

Figure 5: Fraction of consumers who observe exactly one price, conditional on observing at least one price, as a function of n for $N = 10$ and $k = 3$.

Part (ii) of the proposition is straightforward. An increase in search cost causes nothing but a rise in the total resources spent on search by consumers. As a result of this, buyer welfare declines.

6 Search Cost Heterogeneity

In this section, we extend our model to address consumer heterogeneity. There are numerous ways to model consumer heterogeneity, but here we focus on heterogeneity of search costs. There is an ample empirical evidence suggesting that buyers differ in their search costs in real world markets (e.g., Hong and Shum (2006), De los Santos et al. (2012), Honka and Chintagunta (2017)).

The most common way of incorporating search cost heterogeneity to our model is the introduction of consumers who observe all prices in the market for free. With respect to search costs, one may think of these buyers as ones with zero search cost or ones who enjoy shopping (e.g., Stahl (1989), Janssen and Moraga-Gonzalez (2004)). Thus, we assume that $\lambda \in (0, 1)$ is the (exogenously given) share of buyers who observe all prices. Call these *costless buyers*, and refer to the rest of the buyers as *costly buyers*. Since our aim is to show that the main mechanisms of our model are present in the presence of search cost heterogeneity, we will restrict our attention to $\theta_2 > 0$, or $\underline{n} = 2$. The rest of the model and timing of the game are the same as in the main model.

Like in the main model, there is a unique stable SBNE, where costly consumers search actively, for sufficiently small search costs. In the equilibrium, costly buyers randomize over searching $N-1$ and N firms. Then, it is easy to see that a monopolist seller sets price v whereas sellers in a market with $n \geq 3$ price at the production marginal cost. Duopoly sellers play a mixed-strategy. If we let $q \equiv q_{N-1}$ so that $q_N = 1-q$, the expected profit of seller j that sets price p is

$$\Pi_{2j}(p, p_{-j}) = \left[(1-\lambda)\left(\frac{q}{N} + \left(\frac{q(N-2)}{N} + (1-q)\right)x_2(p)\right) + \lambda x_2(p)\right]p.$$

It is easy to establish that the highest price in the support of $x_2(p)$ must be equal to v. Using this fact, we can derive the price distribution:

$$x_2(p) = \mu(q)\left(\frac{v}{p} - 1\right), \text{ with support } \left[\frac{\mu(q)}{1+\mu(q)}v, v\right], \tag{5}$$

where

$$\mu(q) = \frac{q(1-\lambda)}{N - 2q(1-\lambda)}$$

which is the ratio of locked-in consumers to that price comparing buyers (as in Varian (1980) and Stahl (1989)).

Given the above pricing of the sellers, a costly buyer is indifferent between searching $N-1$ and N firms if

$$c = \frac{2\theta_2}{N}\left(E[p] - E[\min\{p_1, p_2\}]\right), \tag{6}$$

where $E[p]$ is the expected price and $E[\min\{p_1, p_2\}]$ is the expected minimum of the two prices in the duopoly market. The following proposition shows that there exists a solution to (6) for sufficiently small search costs.

Proposition 8. *For any $v > 0$, $\underline{n} = 2$ and $\lambda \in (0,1)$, there exists $\bar{c} \in (0, v)$ such that for $c \leq \bar{c}$ there exists a unique stable SBNE given by $((x_n)_{n=1}^{N}, q)$, where*

$$1 - x_1(p) = \begin{cases} 0, & p < v, \\ 1, & p \geq v, \end{cases}$$

$$1 - x_n(p) = \begin{cases} 0, & p < 0, \\ 1, & p \geq 0, \quad \text{for } \; 3 \leq n \leq N, \end{cases}$$

$x_2(p)$ is given by (5), and q is implicitly determined by (6).

Our main concern is the impact of greater product availability on prices and costly consumers' well-being. Like in the previous section, we say that a product is more available if θ_i increases at the expense of θ_j where $i > j$. The following proposition states the main result.

Proposition 9. *In the unique stable SBNE where costly consumers randomize over searching $N-1$ and N firms, an increase in θ_i with a corresponding decrease in θ_j*

(i) decreases the expected price and improves costly consumers' well-being for $j = 1$,

(ii) does not affect the expected price and costly consumers' well-being for $j \geq 3$,

(iii) raises the expected price and harms costly consumers' well-being for $j = 2$.

The intuition is similar to that behind Proposition 5. The only quantitative difference is that, following a greater product availability, the expected price paid by buyers conditional on purchase rises to a lesser extent than in Proposition 5. This is clearly due to the presence of costless buyers who always pay the lowest price in the market.

7 Price Comparison Websites

In this section, we extend our model even further to analyze roles of price comparison websites. Online platforms which aggregate prices serve as a less costly means through which buyers can obtain information about active sellers and their prices (e.g., Brown and Goolsbee (2002), Ellison and Ellison (2009)). We model such markets by allowing costly consumers to search for the product and prices on a price comparison website.

We demonstrate that greater product availability results in a higher expected price and reduces consumer welfare.

We incorporate a price aggregating platform to our model as follows. A costly consumer learns for free whether the product is available at a randomly selected firm and, if it is, what price it charges. After that, the consumer decides whether to access a price aggregator (or simply *search*) at cost c. If she searches, the consumer learns product availability at the rest of the firms and prices (of all active sellers). If she does not access the price aggregator, she can make a purchase from the initial firm whose price she learned, given that the firm has the product, or can drop out of the market in case the firm does not offer the product. Such search protocol is known as *newspaper search* in the literature (e.g., Varian (1980), Dana (1994)).

There are several important differences of the above search protocol from the traditional newspaper search widely used in search literature where consumers know product availability. In the literature, a price aggregator provides information about prices. In our paper, it gives information about both product availability and prices. Specifically, consumers before searching do not know how many prices they will obtain from a price aggregator. Hence, uncertainty about product availability is an essential factor that affect consumers' decision whether or not to search on the price aggregator.

To pinpoint the important mechanisms of the model, we simplify it by assuming that $N = 3$ and $\theta_0 + \theta_1 = 0$. Following the literature on newspaper search, we assume that $\lambda \in (0, 1)$ share of consumers are costless consumers as in the previous section.

The timing of the game is as follows. First, nature chooses sellers in the market according to θ. Firms observe which of them entered the market. Second, sellers simultaneously set prices. Costless consumers observe all prices. Third, without knowing prices and product availability, each consumer observes product availability and a price (if the product is available) of a randomly chosen firm. Fourth, costly consumers choose whether to search or not. Searching consumers observe product availability and prices in the entire market. Consumers who observe at least one price may make purchase.

We employ a Symmetric Reservation Price Equilibrium (SRPE) to solve the game. An SRPE is a Perfect Bayesian-Nash equilibrium, where costly buyers employ a reservation price strategy. The reservation price of a costly consumer, denoted by ρ, is a price at which she is indifferent between buying from the first seller and searching prices on a price aggregator. If a price that a consumer observes before accessing a price aggregator is lower than ρ, she makes a purchase outright; and if it is higher than ρ, the consumers searches. We assume that in equilibrium, a costly consumer buys outright from the first seller even if its price is equal to ρ because, if the consumer observing ρ searches with strictly positive probability sellers can always slightly undercut prices below ρ. Hence, an SRPE is a distribution of prices $(x_n)_{n=2}^{3}$ and the reservation price ρ such that (i) each sellers maximizes its profit given n and strategies of other sellers and consumers, (ii) each costly consumer searches according to the reservation price strategy ρ, (iii) and costly consumers have passive beliefs and update their beliefs according to Bayes' rule whenever possible.

Points (i) and (ii) in the previous paragraph are intuitive. Point (iii) allows a costly consumer to update her belief about realization of n after they learn product availability and price (if the first firm offer the product) of the first firm. Passive beliefs means that, if a costly consumer observes a price that is not a part of an equilibrium, she believes that the other sellers do not deviate. Also when a costly consumer observes a price which the not in the support of both equilibrium x_2 and x_3, she assigns an equal probability to each $n = 2$ and $n = 3$.

7.1 Equilibrium

In the following proposition, we show existence of an SRPE and characterize it.

Proposition 10. *For any $v > 0$, $\theta_0 = 0$, $\theta_1 = 0$, and $\underline{n} = 2$, there exists $\overline{\lambda} \in (0,1)$ such that for $\lambda \geq \overline{\lambda}$ there exists an SRPE given by $((x_n)_{n=2}^{3}, \rho)$ where*

$$x_2(p) = \frac{1-\lambda}{1+2\lambda}\left(\frac{\min\{\rho, v\}}{p} - 1\right), \text{ with support } \left[\left(\frac{1-\lambda}{2+\lambda}\right)\min\{\rho, v\}, \min\{\rho, v\}\right], \tag{7}$$

$$x_3(p) = \left[\frac{1-\lambda}{3\lambda}\left(\frac{\min\{\rho, v\}}{p} - 1\right)\right]^{\frac{1}{2}}, \text{ with support } \left[\left(\frac{1-\lambda}{1+2\lambda}\right)\min\{\rho, v\}, \min\{\rho, v\}\right], \tag{8}$$

and there exists $\overline{c}$ such that for $c \leq \overline{c}$, ρ is determined by

$$\int_{\frac{(1-\lambda)\min\{\rho,v\}}{1+2\lambda}}^{\min\{\rho,v\}} \left(1 - x_3(p)^2\right) dp = c \tag{9}$$

and for $c > \overline{c}$, $\rho = v$.

The intuition behind the proposition is as follows. We first suppose that there exists a unique ρ and derive optimal pricing of sellers. Then, we check whether there exists a unique ρ, given the pricing policy of the sellers.

For a given ρ, we obtain the following properties of price distributions: $x_n(\min\{\rho, v\}) = 0$ and x_n has no mass points or flat regions in its support for $n = 2, 3$. The intuition behind the first point is that, given no other sellers price higher than $\min\{v, \rho\}$, an individual seller that prices greater than $\min\{v, \rho\}$ does not make sales. If $\rho \geq v$, consumers clearly do not buy at a price greater than v. If $\rho < v$ and a seller prices higher than ρ, costly consumers who visit the seller under question search and buy from a rival seller since rival sellers price below ρ. Also consumer who happen to visit one of rival sellers buy outright from them. Costly consumers who happen to visit an inactive firm engage in search because exiting the market without purchase is payoff equivalent to buying at price v. These consumers observe all price and clearly buy from a rival seller that has the lowest price. That x_n is atomless and contains no flat region follows from standard arguments in search literature.

As the next step in the proof, we determine uniqueness of ρ. Note that, if a costly consumer observes price p in the support of x_3, she updates her belief about the probability that there are two sellers, denoted by $\omega(p)$, according Bayes' rule:

$$\omega(p) = \frac{\frac{2}{3}\theta_2 x_2'(p)}{\frac{2}{3}\theta_2 x_2'(p) + \theta_3 x_3'(p)}.$$

Since $x_2'(\rho) = -(1-\lambda)/[(1+2\lambda)\rho]$ and $x_3'(\rho) = -\infty$, we have $\omega(\rho) = 0$. Then, it is easy to see that the reservation price is determined by (9). Observe that the left-hand side of the equation is strictly increasing in ρ. Since the left-hand side of (9) is independent of ρ, there must be a unique solution to the equation if there exists one. If the solution does not exist, we set $\rho = v$.

Finally, we determine conditions on λ under which the above SRPE exists. The reason we do so is that, when a costly consumer observes a price in the range between the lower bound of the support of x_2, which is $(1-\lambda)\min\{\rho, v\}(2+\lambda)$, and that of x_3, which is $(1-\lambda)\min\{\rho, v\}(1+2\lambda)$, at the first seller, she correctly concludes that there are two sellers in the market. Hence, there is a discontinuity in the posterior belief of a costly consumer at the price slightly below the lower bound of the support of x_3. A costly consumer that observes this price does not search, or acts according to the reservation price ρ by making a purchase outright, if

$$\int_{\left(\frac{1-\lambda}{2+\lambda}\right)\min\{\rho,v\}}^{\left(\frac{1-\lambda}{1+2\lambda}\right)\min\{\rho,v\}} (1 - x_2(p)) \leq c.$$

A solution to the above inequality determines the cutoff λ, such that the SRPE exists.

We note that the behavior of costly consumers who observe that the first firm does not have the product correctly conclude that there are two sellers in the market. Yet such consumers do not act differently from consumers who observe price in the support of x_3 at the first firm. The reasoning is as follows. Suppose a costly consumer observes a price equal to ρ at the first firm. She concludes that the market is a triopoly and, thus, indifferent between buying immediately and searching. Now, suppose that this consumer finds that the first firm does not offer the product. Then, we can say that the consumer observes a price equal to v and concludes that the market is a duopoly. This consumer clearly searches if $\rho < v$. If $\rho = v$, the consumer still searches as she has a higher incentive to search than she would have if she knew that the market was a triopoly. As a result, costly consumers who do not find the product at the first firm always search, and such a behavior is consistent with SRPE.

7.2 Impact of Greater Product Availability

Now, we assess an impact of an increase in product availability on consumers' welfare.

Proposition 11. *An increase in θ_3 with a corresponding decrease in θ_2 raises the expected prices and harms consumers.*

The proof is in the appendix, and the intuition is as follows. Notice from equations 7 and 8 that the change in θ (as in the proposition) does not affect pricing policies of the sellers if it does not affect costly consumers' search strategies. It is easy to see that the reservation price of costly consumers in (9) is independent of θ. Then, greater product availability does not change behaviors of both sellers and buyers. However, since the expected price in a duopoly market is lower than that in a triopoly market, the ex-ante expected price rises. This implies that consumers are worse-off simply because they are more likely to face a less competitive market.

The intuition behind why the expected price is higher when there are more sellers is akin to that in traditional models of Varian (1980) and Stahl (1989). As the number of sellers in the market rises, competition for costless buyers intensifies and each sellers' share of locked-in consumers falls. The former impact is higher than the latter so that sellers choose to focus on reaping-off locked in consumers rather than fiercely competing for costless buyers. As a result, the expected price is higher with 3 sellers than with 2 sellers.

8 Conclusion

We see the current paper to be the first to address information asymmetry between buyers and sellers on product availability in search markets. The results of the paper suggest that ignoring such uncertainty in the analysis may be misleading. Importantly, accounting for information uncertainty about product availability may help to better evaluate policy interventions, such as stimulating firm entry. If policy makers ignore this uncertainty, they would expect such policies to lead to greater number of competitors and, therefore, stronger competition. However, if buyers' uncertainty is taken into account, it may reveal that the policy may mitigate consumers' willingness to search, which may lead to softer competition.

We understand that the model is restricted in a sense that it considers a homogeneous goods market. In reality, goods are differentiated in many markets and buyers compare different deals not only on the basis of prices but also other characteristics of products, such as quality or appearance. Therefore, a natural extension of the model would be to consider differentiated goods, as in Perloff and Salop (1985), Anderson et al. (1992), Moraga-Gonzalez et al. (2018). The extension may introduce new mechanisms. For example, the presence of uncertainty on product availability will motivate consumers to search more than in the absence of it. Still, it is ambiguous whether sellers respond to the presence of uncertainty by lowering their prices, as a consumer who observes two prices may still buy from a seller with a higher price because their product has a higher match value than that of the lower-priced seller.

A Proofs

Before we prove Proposition 2, we present a lemma that points out an important relationship between $\alpha_{nk}(x)$ and $\alpha_{nk+1}(x)$.

Lemma 2. $\sum_{m=0}^{l} \alpha_{nk,m} \geq \sum_{m=0}^{l} \alpha_{nk+1,m}$ *for all* $0 \geq l \geq k+1$.

Proof. We prove the lemma with the help of the following three claims.

Claim 1. $\alpha_{nk,0} > \alpha_{nk+1,0}$ *for any* $\underline{n} \leq n \leq N-k$ *and* $\alpha_{nk,1} > \alpha_{nk+1,1}$ *for* $n = N-k+1$.

Proof of Claim 1. It is straightforward to calculate:

$$\alpha_{nk,0} - \alpha_{nk+1,0} = \frac{\binom{N-n}{k}}{\binom{N}{k}} - \frac{\binom{N-n}{k+1}}{\binom{N}{k+1}} = \frac{(N-n)!(N-k-1)!}{N!(N-n-k-1)!}\left(\frac{N-k}{N-n-k} - 1\right) > 0.$$

For $n = N-k+1$, we have

$$\alpha_{nk,1} - \alpha_{nk+1,1} = \frac{\binom{k-1}{k-1}\binom{N-k+1}{1}}{\binom{N}{N-k+1}} - \frac{\binom{k-1}{k}\binom{N-k+1}{1}}{\binom{N}{k+1}} = \frac{\binom{k-1}{k-1}\binom{N-k+1}{1}}{\binom{N}{N-k+1}} > 0,$$

where the second equality is due to the fact that $\binom{k-1}{k} = 0$. $\square$

Claim 2. *Probability series—*$\alpha_{nk,0}, \alpha_{nk,1}, ..., \alpha_{nk,k}$*—is single peaked with* $\arg\max_m \alpha_{nk,m} \leq \arg\max_m \alpha_{nk+1,m}$.

Proof of Claim 2. Note that $\alpha_{nk,m}$ is a probability mass function of a hypergeometric distribution. The function assigns strictly positive probabilities only to $0 \leq m \leq \min\{n,k\}$ and zero probability to other values of m, where m is an integer. It is known that the hypergeometric distribution is unimodal. Then, $\alpha_{nk,m}$ achieves its maximum at the integer above value t_k which satisfies $\alpha_{nk,t_k} = \alpha_{nk,t_{k+1}}$. Clearly, the equality implies

$$\binom{N-n}{k-t_k}\binom{n}{t_k} = \binom{N-n}{k-t_k-1}\binom{n}{t_k+1}.$$

It is easy to check that this can be reduced to

$$(N-n-(k-t_k-1))(t_k+1) = (k-t_k)(n-t_k),$$

which yields

$$t_k = \frac{(k+1)(n+1)}{N+2} - 1. \tag{A.1}$$

Notice that it is possible that $t_k < 0$. Define $M(z)$ to be an operator which spits out z if z is a positive integer, the next high positive integer if z is a positive fraction, and 0 if $z < 0$. Then, the solution to $\arg\max_m \alpha_{nk,m}$ is t_k such that

$$t_k = M(t_k).$$

Similarly, the solution to $\arg\max_m \alpha_{nk+1,m}$ is

$$t_{k+1} = M(t_{k+1}),$$

where, it is easy to check that,

$$t_{k+1} = \frac{(k+1)(n+1)}{N+2}. \tag{A.2}$$

Then, $\arg\max_m \alpha_{nk,m} \leq \arg\max_m \alpha_{nk+1,m}$ holds if

$$t_k = \frac{(k+1)(n+1)}{N+2} - 1 \leq \frac{(k+1)(n+1)}{N+2} = t_{k+1},$$

which is certainly true. This completes the proof. □

Claim 3. *$s_{nk,m} < s_{nk+1,m}$ for any $0 \leq m \leq k$.*

Proof of Claim 3. It is easy to calculate that

$$\begin{aligned} s_{nk,m} &= \frac{\alpha_{nk,m+1}}{\alpha_{nk,m}} = \frac{\binom{N-n}{k-m-1}\binom{n}{m+1}}{\binom{N-n}{k-m}\binom{n}{m}} = \frac{(k-m)(n-m)}{(N-n-(k-m-1))(m+1)}, \\ s_{nk+1,m} &= \frac{\alpha_{nk+1,m+1}}{\alpha_{nk+1,m}} = \frac{\binom{N-n}{k+1-m}\binom{n}{m}}{\binom{N-n}{k-m}\binom{n}{m+1}} = \frac{(k+1-m)(n-m)}{(N-n-(k-m))(m+1)}. \end{aligned}$$

Both $s_{nk,m}$ and $s_{nk+1,m}$ are less than 1 for m close to $\min\{k, n\}$ meaning that the series are decreasing in m for high values of m. Observe that $s_{nk,m} < s_{nk+1,m}$ is true if

$$\frac{k-m}{N-n-(k-m-1)} < \frac{k+1-m}{N-n-(k-m)},$$

which certainly holds because the numerator of the LHS is lower than that of the RHS and the denominator of the LHS is greater than that of the RHS. □

Claim (ii) implies that $\alpha_{nk,m}$ obtains its maximum no later than $\alpha_{nk+1,m}$ in m. Claim (iii) means that when $\alpha_{nk,m}$ is increasing in m, it increases more slowly than $\alpha_{nk+1,m}$; $\alpha_{nk,m}$ starts decreasing in m no later than $\alpha_{nk+1,m}$; and when both $\alpha_{nk,m}$ and $\alpha_{nk,m}$ $\alpha_{nk+1,m}$ decrease in m, the former decreases faster in m than the latter. Along with Claim (i) and the fact that $\sum_{m=0}^{k} \alpha_{nk,m} = 1$, these establish the the proof of the lemma. □

Now we are ready to prove Proposition 2.

A.1 Proof of Proposition 2

Proof of the first part follows from the discussion in the paragraphs after the proposition in the body of our paper.

(ii) For this part, it suffices to show that, in equilibrium, it cannot be that consumers are indifferent over searching $k-1$, k, and $k+1$ firms. The proof is by contradiction. Suppose that consumers are indifferent over searching $k-1$, k, and $k+1$ firms. Also let $k-1(\geq 1)$ be the lowest number such that $q_0 = ... = q_{k-2} = 0$ and $q_{k-1} > 0$. Then, as not buying yields the same payoff as buying at price v,the expected payoff that searching

k firms yields is

$$
\begin{aligned}
& v-(\theta_0+\theta_1)v-\sum_{n=\underline{n}}^{N-k+1}\theta_n\left(\alpha_{nk,0}v-\int_{\underline{p}}^{v}p\alpha_{nk}'(x_n(p))x_n'(p)dp\right)-(k-1)c \\
= \;& v-(\theta_0+\theta_1)v+\sum_{n=\underline{n}}^{N-k+1}\theta_n\left(\alpha_{nk,0}v+\int_0^1 p_n(x_n)\alpha_{nk}'(x_n)dx_n\right)-(k-1)c,
\end{aligned}
$$

where we used the fact that the PDF of the lowest k prices is

$$
\frac{d}{dp}\left(1-x_n(p)^k\right)=-kx_n(p)^{k-1}x_n'(p).
$$

and obtained the second line by changing $p(x_n)$ to x. If a consumer is indifferent between visiting $k-1$ and k firms as well as k and $k+1$ firms, the following set of equations must hold:

$$
\begin{aligned}
\sum_{n=\underline{n}}^{N-k+1}\theta_n\left((\alpha_{nk-1,0}-\alpha_{nk,0})v+\int_0^1 p_n(x_n)\left(\alpha_{nk-1}'(x_n)-\alpha_{nk}'(x_n)\right)dx_n\right)&=c,\\
\sum_{n=\underline{n}}^{N-k+1}\theta_n\left((\alpha_{nk,0}-\alpha_{nk+1,0})v+\int_0^1 p_n(x_n)\left(\alpha_{nk}'(x_n)-\alpha_{nk+1}'(x_n)\right)dx_n\right)&=c.
\end{aligned}
$$

As the RHSs of the equations are equal, we have

$$
\textstyle\sum_{n=\underline{n}}^{N-k+1}\theta_n\left((\alpha_{nk-1,0}+\alpha_{nk+1,0}-2\alpha_{nk,0})v+\int_0^1 p(x_n)\left(\alpha_{nk-1}'(x_n)+\alpha_{nk+1}'(x_n)-2\alpha_{nk}'(x_n)\right)dx_n\right)=0,
$$

or applying integration by parts and the facts that $\alpha_{nk}(0)=\alpha_{nk,0}$ and $\alpha_{nk}(1)=1$, and $p(0)=v$, we obtain

$$
-\sum_{n=\underline{n}}^{N-k+1}\theta_n\int_0^1 p_n'(x_n)\left(\alpha_{nk-1}(x_n)+\alpha_{nk+1}(x_n)-2\alpha_{nk}(x_n)\right)dx_n=0. \tag{A.3}
$$

However, the LHS of the equality is strictly positive for $x\in[0,1]$, which we prove as follows.

We, first, note the values of the LHS in the limits. At $x=0$, the LHS reduces to

$$
\begin{aligned}
\alpha_{nk-1,0}+\alpha_{nk+1,0}-2\alpha_{nk,0}&=\frac{\binom{N-n}{k}}{\binom{N}{k}}\left(\frac{N-(k-1)}{N-n-(k-1)}+\frac{N-n-k}{N-k}-2\right)\\
&=\frac{\binom{N-n}{k}}{\binom{N}{k}}\left[\frac{(N-k)^2+(N-k)+(N-n-(k-1))^2-(N-n-(k-1))-2(N-n-(k-1))(N-k)}{(N-n-(k-1))(N-k)}\right]\\
&=\frac{\binom{N-n}{k}}{\binom{N}{k}}\left[\frac{\Big((N-k)-(N-n-(k-1))\Big)^2+(n-1)}{(N-n-(k-1))(N-k)}\right],
\end{aligned} \tag{A.4}
$$

which is clearly positive. For $x_n=1$, the LHS is equal to zero for any $\underline{n}\leq n\leq N-k+1$.

Let

$$\begin{aligned}\sigma_{nk,m} &\equiv \frac{\alpha_{nk-1,m}+\alpha_{nk+1,m}}{2}\\ &= \frac{\alpha_{nk,m}}{2}\left(\frac{(k-m)(N-(k-1))}{k(N-n-(k-1-m))}+\frac{(k+1)(N-n-(k-m))}{(k+1-m)(N-k)}\right).\end{aligned}$$

Then, if cumulative distribution function $\sum_{m=0}^{l}\alpha_{nk,m}$ second-order stochastically dominates $\sum_{m=0}^{l}\sigma_{nk,m}$, the LHS of (A.3) is indeed strictly positive for $0<x<1$ as x^m is a convex function of m. Noting that $\alpha_{nk,m}$ is a hypergeometric mass probability, it is easy to see the following means

$$\begin{aligned}\mathbb{E}_{\alpha_{nk}}[m] &= \sum_{m=0}^{n}\alpha_{nk,m}m = k\frac{n}{N},\\ \mathbb{E}_{\sigma_{nk}}[m] &= \frac{1}{2}\sum_{m=0}^{n}\left(\alpha_{nk-1,m}+\alpha_{nk+1,m}\right)m = \frac{(k-1+k+1)n}{2N} = k\frac{n}{N}.\end{aligned}$$

Noting that the following relationship between the variances $\mathbb{V}_{\alpha_{nk-1}}[m]+\mathbb{V}_{\alpha_{nk+1}}[m] = 2\mathbb{V}_{\alpha_{nk}}[m]-2n(N-n)/(N^2(N-1))$, we have

$$\begin{aligned}\mathbb{V}_{\alpha_{nk}}[m] &= k\frac{n(N-n)(N-k)}{N^2(N-1)},\\ \mathbb{V}_{\sigma_{nk}}[m] &= \mathbb{E}_{\sigma_{nk}}[m^2]-\left(\mathbb{E}_{\sigma_{nk}}[m]\right)^2\\ &= \frac{1}{2}\left(\mathbb{E}_{\alpha_{nk-1}}[m^2]+\mathbb{E}_{\alpha_{nk+1}}[m^2]\right)-\frac{1}{4}\left(\mathbb{E}_{\alpha_{nk-1}}[m]^2+\mathbb{E}_{\alpha_{nk+1}}[m]^2+2\mathbb{E}_{\alpha_{nk-1}}[m]\mathbb{E}_{\alpha_{nk+1}}[m]\right)\\ &= \frac{1}{2}\left(\mathbb{V}_{\alpha_{nk-1}}[m]+\mathbb{V}_{\alpha_{nk+1}}[m]\right)+\frac{1}{4}\left(\mathbb{E}_{\alpha_{nk-1}}[m]^2+\mathbb{E}_{\alpha_{nk+1}}[m]^2-2\mathbb{E}_{\alpha_{nk-1}}[m]\mathbb{E}_{\alpha_{nk+1}}[m]\right)\\ &= \mathbb{V}_{\alpha_{nk}}[m]-\frac{n(N-n)}{N^2(N-1)}+\frac{n^2}{N^2}.\end{aligned}$$

Notice that the latter variance is greater than the former if $\frac{n^2}{N^2}>\frac{n(N-n)}{N^2(N-1)}$ or $n>\frac{N-n}{N-1}$, which is certainly true for $n\geq 2$. Hence, $\sum_{m=0}^{l}\sigma_{nk,m}$ is a mean-preserving spread function of $\sum_{m=0}^{l}\alpha_{nk,m}$ for $n\geq 2$, which implies $\sum_{l=0}^{l}\sum_{m=0}^{l}\sigma_{nk,m}>\sum_{l=0}^{l}\sum_{m=0}^{l}\alpha_{nk,m}$ for $l\leq n-1$ i.e., the second-order stochastic dominance of the latter over the former. As x_n^m is a decreasing convex function of m for $0<x_n<1$, it must be $\sum_{m=0}^{n}x_n^m\sigma_{nk,m}>\sum_{m=0}^{n}x_n^m\alpha_{nk,m}$, or $\sigma_{nk}(x)-\alpha_{nk}(x)<0$, which proves that the LHS of (A.3) is strictly positive, a contradiction. This proves that, in equilibrium, consumers cannot be indifferent among searching $k-1$, k, and $k+1$ firms. This completes the proof of the proposition.

A.2 Proofs of Proposition 3

We use the following claim to prove the proposition.

Lemma 3. *P_k-P_{k+1} is positive and strictly concave in $q_k\in(0,1)$.*

Proof of Lemma 3. That P_k-P_{k+1} is positive follows directly from the stochastic dominance in Lemma 2. To show that $P_k - P_{k+1}$ is strictly concave, we note that

$$\frac{d(P_k - P_{k+1})}{dq_k} = \upsilon \sum_{n=\underline{n}}^{N-k+1} \theta_n \int_0^1 \frac{(\alpha'_{nk}(x_n) - \alpha'_{nk+1}(x_n))\left(\alpha_{nk,1}\alpha'_{nk+1}(x_n) - \alpha_{nk+1,1}\alpha'_{nk}(x_n)\right)}{\left[\beta'_{nk}(x_n)\right]^2} dx_n,$$

and

$$\frac{d^2(P_k - P_{k+1})}{dq_k^2} = -\upsilon \sum_{n=\underline{n}}^{N-k+1} \theta_n \int_0^1 \frac{2\beta'_{nk}(x_n)(\alpha'_{nk}(x_n) - \alpha'_{nk+1}(x_n))^2\left(\alpha_{nk,1}\alpha'_{nk+1}(x_n) - \alpha_{nk+1,1}\alpha'_{nk}(x_n)\right)}{\left[\beta'_{nk}(x_n)\right]^4} dx_n,$$

which is negative if $\Psi_n(x) \equiv \alpha_{nk,1}\alpha'_{nk+1}(x_n) - \alpha_{nk+1.1}\alpha'_{nk}(x_n) \geq 0$ for each $n \geq \underline{n}$ and $x \in (0,1)$. To simplify the notation, we write x to imply x_n unless stated otherwise. Note that $\Psi_n(0) = \alpha_{nk,1}\alpha_{nk+1,1} - \alpha_{nk+1,1}\alpha_{nk,1} = 0$. Then, $\Psi_n(x) > 0$ for $x \in (0,1)$ if $d\Psi_n(x)/dx > 0$ for $x \in (0,1)$. To show that the derivative is positive, we first note that $\Psi_n(x)$ is $\bar{l} \equiv \min\{n-1, k\}$ times differentiable in x. Second, we point out that lth (where $l \leq \bar{l}$) derivative of the function is

$$\begin{aligned}
\frac{d^l\Psi_n(x)}{dx^l} &= \alpha_{nk,1}\frac{d^l\alpha'_{nk+1}(x)}{dx^l} - \alpha_{nk+1,1}\frac{d^l\alpha'_{nk}(x)}{dx^l}\frac{d^l}{dx^l}\alpha'_{nk}(x) \\
&= \sum_{m=l}^{k+1}\frac{m!}{(m-l)!}\alpha_{nk+1,m}x^{m-l} - \sum_{m=l}^{k}\frac{m!}{(m-l)!}\alpha_{nk,m}x^{m-l}.
\end{aligned} \tag{A.5}$$

Third, we prove that $\frac{d^l\Psi_n(x)}{dx^l} > 0$ for each l such that $1 \leq l \leq \bar{l}$. We start by considering $\bar{l}$'th derivative of $\Psi_n(x)$. For $n-1 \geq k$, we have

$$\frac{d^k\Psi_n(x)}{dx^k} = \alpha_{nk,1}(k+1)!\alpha_{nk+1,k+1} > 0.$$

This means that $d^{k-1}\Psi_n(x)/dx^{k-1}$ in (A.5) is increasing in x. Then, however, it must be that $d^{k-1}\Psi_n(x)/dx^{k-1} > 0$ if it holds for $x = 0$. It is easy to see that

$$\left.\frac{d^{k-1}\Psi_n(x)}{dx^{k-1}}\right|_{x=0} = \alpha_{nk,1}k!\alpha_{nk+1,k} - \alpha_{nk+1,1}k!\alpha_{nk,k},$$

which is positive if $\alpha_{nk,1}\alpha_{nk+1,k} \geq \alpha_{nk+1,1}\alpha_{nk,k}$, which expands to

$$\frac{\binom{N-n}{k+1-1}\binom{n}{1}\binom{N-n}{k-k}\binom{n}{k}}{\binom{N}{k}\binom{N}{k+1}} \leq \frac{\binom{N-n}{k-1}\binom{n}{1}\binom{N-n}{k+1-k}\binom{n}{k}}{\binom{N}{k}\binom{N}{k+1}} \Rightarrow \binom{N-n}{k} \leq \binom{N-n}{k-1}(N-n),$$

or,

$$\binom{N-n}{k} \leq \binom{N-n}{k}\frac{(N-n)k}{N-n-(k-1)}.$$

This is true if $N-n-(k-1) \leq (N-n)k$. The last inequality can be simplified as $0 \leq (N-n+1)(k-1)$ which is clearly true. This shows that (A.5) for $l = k-1$ is positive for $x = 0$. Then, (A.5) for $l = k-1$ is strictly positive for any $x \in (0,1)$.

Now, we repeat similar steps to show that (A.5) holds for $l = k-2$. Namely, the fact that $d^{k-1}\Psi_n(x)/dx^{k-1} > 0$ for $x \in (0,1)$ means that $d^{k-2}\Psi_n(x)/dx^{k-2}$ is strictly increasing in $x \in (0,1)$. Then, $d^{k-2}\Psi_n(x)/dx^{k-2} > 0$ for $x \in (0,1)$ if it is positive for $x = 0$, or

$d^{k-2}\Psi_n(x)/dx^{k-2}|_{x=0} \geq 0$. Instead of proving the inequality each time, we demonstrate that this holds for each l, which is true if $\alpha_{nk+1,1} l! \alpha_{nk,l} \leq \alpha_{nk,1} l! \alpha_{nk+1,l}$, or

$$\alpha_{nk+1,1}\alpha_{nk,l} \leq \alpha_{nk,1}\alpha_{nk+1,l}. \tag{A.6}$$

The inequality can be expanded as

$$\frac{\binom{N-n}{k}\binom{n}{1}\binom{N-n}{k-l}\binom{n}{l}}{\binom{N}{k}\binom{N}{k+1}} \leq \frac{\binom{N-n}{k-1}\binom{n}{1}\binom{N-n}{k+1-l}\binom{n}{l}}{\binom{N}{k}\binom{N}{k+1}},$$

which implies

$$\binom{N-n}{k}\binom{N-n}{k-l} \leq \binom{N-n}{k-1}\binom{N-n}{k+1-l},$$

or,

$$\binom{N-n}{k}\binom{N-n}{k+1-l}\left(\frac{k-l+1}{N-n-(k-l)}\right) \leq \binom{N-n}{k}\binom{N-n}{k+1-l}\left(\frac{k}{N-n-(k-1)}\right).$$

This reduces to $\frac{k-l+1}{N-n-(k-l)} \leq \frac{k}{N-n-(k-1)}$. The last inequality clearly holds as the numerator of the LHS is not greater than that of the RHS, and the denominator of the LHS is not smaller than that of the RHS. This proves that (A.6) holds for any l such that $1 \leq l \leq k$, which in its turn proves that (A.5) is positive for each l, including $l = 1$. Then, it means that $\Psi_n(x)$ is increasing in x. Since $\Psi_n(0) = 0$, it follows that $\Psi_n(x) > 0$ for $x \in (0, 1)$.

Now, it is left to consider the case where $n-1 < k$. Like in the previous case, it suffices to show that $d\Psi_n(x)/dx > 0$ for $x \in (0, 1)$ since $\Psi_n(0) = 0$. For that, we apply the same method as for the case of $n-1 \geq k$. First, we note that if each of l'th $(2 \leq l \leq n-1)$ derivative of $\Psi_n(x)$ with respect to x is positive, then $d\Psi_n(x)/dx > 0$. Second, we note that

$$\frac{d^{n-1}}{dx^{n-1}}\Psi_n(x) = \frac{\binom{N-n}{k-1}\binom{n}{1}\binom{N-n}{k+1-n}\binom{n}{n}}{\binom{N}{k}\binom{N}{k+1}}n! - \frac{\binom{N-n}{k}\binom{n}{1}\binom{N-n}{k-n}\binom{n}{n}}{\binom{N}{k}\binom{N}{k+1}}n!$$

which is strictly positive if

$$\binom{N-n}{k}\binom{N-n}{k+1-n}\left(\frac{k}{N-n-(k-1)}\right) > \binom{N-n}{k}\binom{N-n}{k+1-n}\left(\frac{k-n+1}{N-k}\right),$$

or $\frac{k}{N-n-(k-1)} > \frac{k-n+1}{N-k}$. Clearly, the inequality holds as the numerator of the LHS is greater than that of the RHS and the denominator of the LHS is smaller than that of the RHS. This demonstrates that $\frac{d^{n-1}}{dx^{n-1}}\Psi_n(x) > 0$. The following step is to show that $\frac{d^l}{dx^l}\Psi_n(x) > 0$ and $\frac{d^l}{dx^l}\Psi_n(x)\mid_{x=0} \geq 0$ for each l such that $2 \leq l \leq n-2$, which proves that $\frac{d^l}{dx^l}\Psi_n(x) > 0$. The former is certainly true which follows from the proof of (A.6). This proves that $d\Psi_n(x)/dx > 0$ for $x \in (0, 1)$ because $\Psi_n(0) = 0$. The proof of the lemma is now complete. □

Now, for the proof of the proposition, we first prove existence of $\underline{c}_{k,k+1}$ and $\overline{c}_{k,k+1}$ as well as $\underline{c}_{k,k+1} < \overline{c}_{k,k+1}$ such that (2) holds for $c \in (\underline{c}_{k,k+1}, \overline{c}_{k,k+1})$. From Lemma 3, it follows that $\underline{c}_{k,k+1} = \min\left\{\lim_{q_k \downarrow 0}(P_k - P_{k+1}), \lim_{q_k \uparrow 0}(P_k - P_{k+1})\right\}$ and $\overline{c}_{k,k+1} = \max_{q_k}\{P_k - P_{k+1}\}$. Due to strict concavity of $P_k - P_{k+1}$, we have $0 \leq \underline{c}_{k,k+1} < \overline{c}_{k,k+1}$.

Next, we show that buyers' participation constraint is satisfied, i.e., buyers prefer searching k and $k+1$ firms to not searching at all. This is certainly the case if $P_k + (k-1)\bar{c}_{k,k+1} \leq v$. We fix pricing strategies of firms in an SBNE for $c = \bar{c}_{k,k+1}$ and note that

$$\begin{aligned}
P_k + (k-1)\bar{c}_{k,k+1} &< P_{k-1} + (k-2)\bar{c}_{k,k+1} < ... \\
&< P_1 = (\theta_0 + \theta_1)v + \sum_{n=\underline{n}}^{N-k+1} \theta_n \left(\alpha_{n1,0} v - \alpha_{n1,0} \int_{\underline{p}_n}^{v} p x_n'(p) dp \right) \\
&< v.
\end{aligned}$$

This proves that buyers' participation constraint is satisfied.

Finally, we demonstrate that $\underline{c}_{k,k+1} = 0$ for $k = 1$ and $k = N - \underline{n} + 1$. For that, it suffices to prove that $P_1 - P_2$ and $P_{N-\underline{n}+1} - P_{N-\underline{n}+1}$ converge to zero as $q_1 \to 1$ and $q_{N-\underline{n}+2} \to 1$, respectively. Since $\alpha_{n1}'(x_n) = \alpha_{n1,1}$, we have

$$\begin{aligned}
\lim_{q_1 \uparrow 1}(P_1 - P_2) = & \lim_{q_1 \uparrow 1} \sum_{n=2}^{N} \theta_n v \left((\alpha_{n1,0} - \alpha_{n2,0}) + \int_0^1 \beta_{n1,1} \frac{(\alpha_{n1}'(x_n) - \alpha_{n2}'(x_n))}{\beta_{n1}'(x_n)} dx_n \right) \\
= & \sum_{n=2}^{N} \theta_n v \left((\alpha_{n1,0} - \alpha_{n2,0}) + \int_0^1 (\alpha_{n1}'(x_n) - \alpha_{n2}'(x_n))\, dx_n \right) = 0.
\end{aligned}$$

Here, the first equality in the second line follows from the facts that

$$\begin{aligned}
\lim_{q_1 \uparrow 1} \beta_{n1}'(x_n) &= \lim_{q_1 \uparrow 1} [q_1 \alpha_{n1,1} + (1-q_1) q_2 (\alpha_{n2,1} + \alpha_{n2,2} x_n)] = \alpha_{n1,1}, \\
\lim_{q_1 \uparrow 1} \beta_{n1,1} &= \alpha_{n1,1},
\end{aligned}$$

while the second equality in the second line follows from facts that $\alpha_{n1}(1) = \alpha_{n2}(1) = 1$, $\alpha_{n1}(0) = \alpha_{n1,0}$, and $\alpha_{n2}(0) = \alpha_{n2,0}$. To evaluate $P_{N-\underline{n}+1} - P_{N-\underline{n}+1}$ as $q_{N-\underline{n}+2} \uparrow 1$, we start noting that, in an SBNE where buyers randomize between visiting $N - \underline{n} + 1 (= k)$ and $N - \underline{n} + 2 (= k+1)$ firms, only sellers in a market with $\underline{n}$ sellers play mixed strategy pricing. Also as $\alpha_{\underline{n}k,0} = \alpha_{\underline{n}k+1,0} = \alpha_{\underline{n}k+1,1} = 0$ meaning that $\lim_{q_{k+1} \uparrow 1} \beta_{\underline{n}k,1} = 0$ for $k = N - \underline{n} + 1$, it follows that

$$\begin{aligned}
\lim_{q_{k+1} \uparrow 1}(P_k - P_{k+1}) &= \lim_{q_{k+1} \uparrow 1} \theta_{\underline{n}} v \left(\alpha_{\underline{n}k,0} - \alpha_{\underline{n}k+1,0} + \int_0^1 \frac{\beta_{\underline{n}k,1} \left(\alpha_{\underline{n}k}'(x_{\underline{n}}) - \alpha_{\underline{n}k+1}'(x_{\underline{n}}) \right)}{\beta_{\underline{n}k}'(x_2)} dx_{\underline{n}} \right) \\
&= 0.
\end{aligned}$$

Then, it is indeed that $\underline{c}_{k,k+1} = 0$ for $k = 1$ and $k = N - \underline{n} + 1$.

The proof of the Proposition is complete.

A.3 Proof of Proposition 4

We first prove the existence of the cutoff values of search cost. In the proof of Proposition 2, we showed that the benefit of searching kth firm is greater than that of searching $k+1$th firm. This means that $P_{k-1} + P_{k+1} - 2P_k > 0$. Due to strict inequality, it follows that there must exist a range of value of c such that (4) holds. Moreover, it must be that

$$\begin{aligned} \underline{c}_k &\equiv (P_k - P_{k+1} | q_k = 1), \\ \overline{c}_k &\equiv (P_{k-1} - P_k | q_k = 1). \end{aligned} \tag{A.7}$$

with $0 \leq \underline{c}_k < \overline{c}_k < v$.

We next show that buyers' participation constraint is satisfied. This is true if $v \geq P_k + (k-1)\overline{c}_k$. For given pricing policies of sellers in an SBNE for $c = \overline{c}_k$, we have

$$\begin{aligned} P_k + (k-1)\overline{c}_k &< P_{k-1} - (k-2)\overline{c}_k < \ldots \\ &< P_1 = (\theta_0 + \theta_1)v + \sum_{n=\underline{n}}^{N-k+1} \theta_n \left(\alpha_{n1,0} v + \alpha_{n1,1} \int_{\underline{p}_n}^{v} p x_n'(p) dp \right) \\ &< v. \end{aligned}$$

Thus, the consumers' participation constraint is indeed satisfied. This completes the proof of the proposition.

A.4 Proof of Corollary 1

It suffices to prove that (i) $\underline{c}_k > 0$ for any $2 \leq k \leq N - \underline{n} + 1$, (ii) $P_{N-\underline{n}+1} - P_{N-\underline{n}+2}$ is increasing with $q_{N-\underline{n}+1}$ in the neighborhood of 0 and (iii) $P_1 - P_2$ is decreasing with q_1 in the neighborhood of 0.

For (i), we note that $\underline{c}_k$ is equal to $P_k - P_{k+1}$ evaluated at $q_k \to 1$ in an SBNE where consumers randomize over searching k and $k+1$ firms. Then, it suffices to show that $\lim_{q_k \uparrow 1} P_k - P_{k+1} > 0$ for any $2 \leq k \leq N - \underline{n} + 1$. It is easy to see that

$$\lim_{q_k \to 1} (P_k - P_{k+1}) = - \lim_{q_k \to 1} \sum_{n=\underline{n}}^{N-k+1} \theta_n \int_0^1 p_n'(x_n) \left(\alpha_{nk}(x_n) - \alpha_{nk+1}(x_n) \right) dx_n.$$

As $p_n'(x_n) < 0$, the limiting expression is strictly positive if $\alpha_{nk}(x_n) - \alpha_{nk+1}(x_n) > 0$ for some n such that $\underline{n} \leq n \leq N - k + 1$. For $n = \underline{n}$, the inequality reduces to $\alpha_{\underline{n}k}(x) - \alpha_{\underline{n}k+1}(x) > 0$, which certainly holds due to the stochastic dominance in Lemma 2.

For (ii), we recall from Lemma 3 that $P_{N-\underline{n}+1} - P_{N-\underline{n}+2}$ is positive and strictly concave in $q_{N-\underline{n}+1}$. Also from the proof of Proposition 3, we know that

$$\lim_{q_{N-\underline{n}+1} \downarrow 0} (P_{N-\underline{n}+1} - P_{N-\underline{n}+2}) = 0.$$

These two observations mean that $P_{N-\underline{n}+1} - P_{N-\underline{n}+2}$ must be increasing with $q_{N-\underline{n}+1}$ in the neighborhood of 0.

For (iii), we again recall from Lemma 3 that $P_1 - P_2$ is positive and strictly concave in q_1. In addition, we know from the proof of Proposition 3 that

$$\lim_{q_1 \uparrow 1} (P_1 - P_2) = 0.$$

These two facts imply that $P_1 - P_2$ must be decreasing in q_1 in the neighborhood of 1.

Points (i), (ii), and (iii) establish the proof of the corollary.

A.5 Proof of Proposition 5

We first note that a locally stable SBNE in mixed strategies of consumers is a unique locally stable SBNE for $c \in (0, \min\{\underline{c}_2, \underline{c}_{N-\underline{n}+1}\})$. That the lower cutoff value of the search cost is zero follows from Corollary 1.

Second, we observe that for any realization of $n > \underline{n}$ sellers price at the production marginal cost in equilibrium. To see that, replace k by $N - \underline{n} + 1$ such that $N - k + 2 = N - (N - \underline{n} + 1) + 2 = \underline{n} + 1$. From Proposition 3, it follows that sellers in a market with at least $N - k + 2 = \underline{n} + 1$ number of sellers price at the marginal cost of production. Parts (i) and (ii) of the proposition directly follows from these facts.

For the proof of part (iii) of the proposition, we note that $P_k = (\theta_0 + \theta_1)v + \theta_{\underline{n}} \int_0^1 p_{\underline{n}}(x)\alpha'_{nk}(x)dx$, where $p_{\underline{n}}(x) = q_k \alpha_{\underline{n}k,1} v / \beta'_{\underline{n}k}(x_{\underline{n}})$ and $k = N - \underline{n} + 1$. Similarly, $P_{k+1} = (\theta_0 + \theta_1)v + \theta_{\underline{n}} \int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx$. Therefore, the indifference condition of an individual consumer is given by

$$\theta_{\underline{n}} \int_0^1 p_{\underline{n}}(x) \left(\alpha'_{nk}(x) - \alpha'_{nk+1}(x)\right) dx = c. \tag{A.8}$$

Next, buyers' total outlay is equal to

$$q_k P_k + (1 - q_k) P_{k+1} + (k + 1 - q_k)c = (\theta_0 + \theta_1)v + \theta_{\underline{n}} \int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx + (k+1)c,$$

where we used (A.8) to obtain the equality. The change in the outlay due to a lower product availability, which is associated with an increase in $\theta_{\underline{n}}$, is given by

$$\begin{aligned}\frac{d\left((\theta_0 + \theta_1)v + \theta_{\underline{n}} \int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx + (k+1)c\right)}{d\theta_{\underline{n}}} &= \int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx \\ &+ \theta_{\underline{n}} \frac{\partial \int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx}{\partial q_k} \times \frac{dq_k}{d\theta_{\underline{n}}}.\end{aligned}$$

Since in equilibrium it must be that

$$\begin{aligned}\frac{d\theta_{\underline{n}} \int_0^1 p_{\underline{n}}(x)\left(\alpha'_{nk}(x) - \alpha'_{nk+1}(x)\right)dx}{d\theta_{\underline{n}}} &= \int_0^1 p_{\underline{n}}(x)\left(\alpha'_{nk}(x) - \alpha'_{nk+1}(x)\right)dx \\ &+ \theta_{\underline{n}} \frac{\partial \int_0^1 p_{\underline{n}}(x)\left(\alpha'_{nk}(x) - \alpha'_{nk+1}(x)\right)dx}{\partial q_k} \times \frac{dq_k}{d\theta_{\underline{n}}} = 0,\end{aligned}$$

the change in the buyers' outlay can be rewritten as

$$\int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx - \theta_{\underline{n}} \frac{\partial \int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx}{\partial q_k} \left(\frac{\int_0^1 p_{\underline{n}}(x)\left(\alpha'_{nk}(x) - \alpha'_{nk+1}(x)\right)dx}{\theta_{\underline{n}} \frac{\partial \int_0^1 p_{\underline{n}}(x)\left(\alpha'_{nk}(x) - \alpha'_{nk+1}(x)\right)dx}{\partial q_k}} \right).$$

This can be rewritten as

$$\frac{\partial \int_0^1 p_{\underline{n}}(x)\alpha'_{nk}(x)dx}{\partial q_k} \int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx - \frac{\partial \int_0^1 p_{\underline{n}}(x)\alpha'_{nk+1}(x)dx}{\partial q_k} \int_0^1 p_{\underline{n}}(x)\alpha'_{nk}(x)dx.$$

The expression is negative if

$$\int_0^1 \frac{\alpha_{nk,1}\alpha'_{nk}(x)\alpha'_{nk+1}(x)}{\left[\beta'_{nk}(x)\right]^2}dx \int_0^1 \frac{q_k\alpha_{nk,1}\alpha'_{nk+1}(x)}{\beta'_{nk}(x)}dx - \int_0^1 \frac{\alpha_{nk,1}\left[\alpha'_{nk+1}(x)\right]^2}{\left[\beta'_{nk}(x)\right]^2}dx \int_0^1 \frac{q_k\alpha_{nk,1}\alpha'_{nk}(x)}{\beta'_{nk}(x)}dx < 0,$$

or

$$\int_0^1 \frac{\alpha'_{nk}(x)\alpha'_{nk+1}(x)}{\left[\beta'_{nk}(x)\right]^2}dx \int_0^1 \frac{\alpha'_{nk+1}(x)}{\beta'_{nk}(x)}dx - \int_0^1 \frac{\left[\alpha'_{nk+1}(x)\right]^2}{\left[\beta'_{nk}(x)\right]^2}dx \int_0^1 \frac{\alpha'_{nk}(x)}{\beta'_{nk}(x)}dx < 0.$$

Letting $h \equiv \alpha'_{nk}(x)/\beta'_{nk}(x)$, implying $(1-q_k h)/(1-q_k) = \alpha'_{nk+1}(x)/\beta'_{nk}(x)$, rewrite the inequality as

$$\int_0^1 h(1-q_k h)dx \int_0^1 (1-q_k h)dx - \int_0^1 (1-q_k h)^2 dx \int_0^1 h dx < 0.$$

This simplifies to

$$-\int_0^1 h^2 dx + \left(\int_0^1 h dx\right)^2 < 0,$$

which is clearly true by Cauchy-Schwarz Inequality. This proves buyers' outlay decreases with lower product availability; or equivalently, greater product availability harms buyers.

As a final step, we show that the expected price must increase with greater product availability. Notice that no buyer drops out of the market for any realization of $n \geq \underline{n}$. Also following an increase in the product availability, buyers economize on their search costs as they search less. Hence, the only reason why buyers are worse-off due to greater product availability is that the expected price they pay must rise. This completes the proof.

A.6 Proof of Proposition 6

Clearly, a change in search cost does not directly affect sellers. Hence, any changes in market outcomes caused by search cost must be through buyers' willingness to search. Recall that, in a stable equilibrium, $P_k - P_{k-1}$ is increasing in q_k. Then, an increase in search cost, which must be accompanied with an increase in $P_k - P_{k-1}$, must raise q_k.

To see that buyers' welfare falls with an increase in c, first note that in equilibrium it must be

$$\frac{d(P_k - P_{k-1})}{dc} - 1 = 0. \tag{A.9}$$

Second, the change in the average expected virtual price, denoted by P, due to an increase in c can be written as

$$\begin{aligned}\frac{dP}{dc} &= \frac{d\left(P_{k+1} + q_k(P_k - P_{k+1})\right)}{dc} = \left(\frac{\partial P_{k+1}}{\partial q_k} + P_k - P_{k+1}\right)\frac{dq_k}{dc} + q_k\frac{d(P_k - P_{k-1})}{dc} \\ &= \left(\frac{\partial P_{k+1}}{\partial q_k} + c\right)\frac{dq_k}{dc} + q_k,\end{aligned}$$

where the second line is due to (2) and (A.9). Then, the corresponding change in consumer welfare is

$$\begin{aligned}\frac{d(v - P - [q_k(k-1) + (1-q_k)k]\,c)}{dc} &= -\left(\frac{\partial P_{k+1}}{\partial q_k} + c\right)\frac{dq_k}{dc} - q_k - (k - q_k) + \frac{dq_k}{dc}c \\ &= -\left(\frac{\partial P_{k+1}}{\partial q_k}\right)\frac{dq_k}{dc} - k.\end{aligned}$$

As $dq_k/dc > 0$, the derivative is negative if $\partial P_{k+1}/\partial q_k \geq 0$, or

$$v \sum_{n=\underline{n}}^{N-k+1} \theta_n \int_0^1 \frac{\alpha'_{nk+1}(x)\left(\alpha_{nk,1}\alpha'_{nk+1}(x) - \alpha_{nk+1,1}\alpha'_{nk}(x)\right)}{[\beta'_{nk}(x)]^2} dx \geq 0.$$

However, we know from the proof of Proposition 3 that $\alpha_{nk,1}\alpha'_{nk+1}(x) - \alpha_{nk+1,1}\alpha'_{nk}(x)$ is positive for each n such that $\underline{n} \leq n \leq N - k + 1$, which means that the virtual price for n sellers market is indeed increasing in q_k. This shows that dP_{k+1} is increasing in q_k, meaning that the derivative of consumer welfare w.r.t. c is decreasing.

The proof is complete.

A.7 Proof of Proposition 7

(i) We prove that the average expected price paid conditional on buying decreases as the product becomes more available. This, along with the fact that with greater product availability as the share of consumers who does not make purchase (because they do not find the product) decreases, will mean that buyers' welfare improves when product becomes more available.

Suppose buyers search k firms. Then, the expected price paid by buyers conditional on their observing at least one price in a market with n sellers is (recall (3))

$$\frac{\alpha_{nk,1}}{1 - \alpha_{nk,0}} v.$$

To prove (a), we need to show that the fraction $\alpha_{nk,1}/(1 - \alpha_{nk,0})$ is decreasing in n such that $1 \leq n \leq N - k + 1$, or

$$\frac{\alpha_{nk,1}}{1 - \alpha_{nk,0}} - \frac{\alpha_{n+1k,1}}{1 - \alpha_{n+1k,0}} > 0.$$

Observe that the inequality certainly holds for $\alpha_{nk,1} \geq \alpha_{n+1k,1}$ because $\alpha_{nk,0} > \alpha_{n+1k,0}$ (which is easy to check).

Assume that $\alpha_{nk,1} < \alpha_{n+1k,1}$. Using the definition of $\alpha_{nk,m}$ and simplifying, it is easy to show that $\alpha_{nk,1} < \alpha_{n+1k,1}$ translates into $N - nk - k + 1 > 0$. Next, simplify the inequality to be proven as

$$\frac{(1 - \alpha_{n+1k,0})\alpha_{nk,1} - (1 - \alpha_{nk,0})\alpha_{n+1k,0}}{(1 - \alpha_{nk,0})(1 - \alpha_{n+1k,0})} > 0.$$

The inequality holds if the numerator of its left-hand side is positive:

$$(1 - \alpha_{n+1k,0})\alpha_{nk,1} - (1 - \alpha_{nk,0})\alpha_{n+1k,0} > 0.$$

Employing the definition of $\alpha_{nk,m}$ expand the LHS of the inequality as follows:

$$\left[1-\frac{\binom{N-n-1}{k}}{\binom{N}{k}}\right]\frac{\binom{N-n}{k-1}n}{\binom{N}{k}}-\left[1-\frac{\binom{N-n}{k}}{\binom{N}{k}}\right]\frac{\binom{N-n-1}{k-1}(n+1)}{\binom{N}{k}}>0,$$

or

$$\left[\binom{N}{k}-\binom{N-n-1}{k}\right]\binom{N-n}{k-1}n-\left[\binom{N}{k}-\binom{N-n}{k}\right]\binom{N-n-1}{k-1}(n+1)>0,$$

Since

$$\binom{N-n-1}{k}=\binom{N-n}{k}\frac{N-n-k}{N-n},\quad \binom{N-n}{k-1}=\binom{N-n-1}{k-1}\frac{N-n}{N-n-k-1},$$

simplify the inequality as

$$-\binom{N}{k}(N-nk-k+1)+\binom{N-n}{k}(N-k+1)>0.$$

Divide both sides of the inequality by $N-nk-k+1(>0)$ and $N-k+1$ and rearrange to obtain

$$\frac{\binom{N-n}{k}}{N-nk-k+1}>\frac{\binom{N}{k}}{N-k+1},$$

or

$$\frac{(N-n)!}{(N-n-k)!(N-nk-k+1)}>\frac{N!}{(N-k)!(N-k+1)}. \tag{A.10}$$

Observe that for $n=0$, the LHS and the RHS of the inequality are equal to each other. Then, the inequality holds for all $1\le n\le N-k+1$ if the LHS is increasing in n. To show that, we take the derivative of the LHS with respect to n and show that it is positive. As n is an integer, we apply Gamma function to take the derivative. First, we rewrite the LHS as

$$\frac{\Gamma(N-n+1)}{\Gamma(N-n-k+1)(N-nk-k+1)},$$

where Γ stands for Gamma function such that $\Gamma(x+1)=x!$. Noting that, for positive integer x,

$$\frac{d\Gamma(x+1)}{dx}=x!\left(-\gamma+\sum_{l=1}^{x}\frac{1}{l}\right),$$

where $\gamma=\lim_{x\to\infty}\left(-\ln(x)+\sum_{l=1}^{x}\frac{1}{l}\right)$ is the Euler-Mascheroni constant, the derivative of the LHS of the inequality is

$$\frac{(N-n-k)!(N-n)!\left\{-(N-nk-k+1)\left(-\gamma+\sum\limits_{l=1}^{N-n}\frac{1}{l}\right)+(N-nk-k+1)\left(-\gamma+\sum\limits_{l=1}^{N-n-k}\frac{1}{l}\right)+k\right\}}{[\Gamma(N-n-k+1)(N-nk-k+1)]^2}.$$

The derivative is positive if the term in the curly brackets in the numerator is positive, or $k - (N - nk - k + 1) \sum_{l=N-n-k+1}^{N-n} (1/l) > 0$. This is true as

$$\begin{aligned}\frac{k}{N-nk-k+1} &> \frac{k}{N-n-k+1} = \frac{1}{N-n-k+1} + \frac{1}{N-n-k+1} + ... + \frac{1}{N-n-k+1} \\ &> \frac{1}{N-n-k+1} + \frac{1}{N-n-k+2} + ... + \frac{1}{N-n} = \sum_{l=N-n-k+1}^{N-n} \frac{1}{l}.\end{aligned}$$

This demonstrates that the derivative of the LHS of (A.10) is positive. This, in its turn, implies that (A.10) is true for $n \geq 1$ and $N - nk - k + 1 > 0$, meaning that the expected price paid conditional on observing at least one price is decreasing with n for $\alpha_{nk,1} < \alpha_{n+1k,1}$. Then, the average expected price paid conditional on buying is decreasing with greater product availability.

To show that buyers' welfare increases with greater product availability, it suffices to demonstrate that the expected virtual price falls as the product becomes more available. The latter statement is true if the expected virtual price in a market with n sellers, P_k^n, decreases with n for $1 \leq n \leq N - k + 1$. Observe that

$$P_k^n = \alpha_{nk,0} v + (1 - \alpha_{nk,0}) \frac{\alpha_{nk,1}}{1 - \alpha_{nk,0}} v,$$

or the expected virtual price in a market with n sellers is a weighted average of the the monopoly price v and the expected price paid by buyers conditional on observing at least one price. First, it is easy to check that $\alpha_{nk,0}$ is decreasing in n. This means that monopoly price receives less weight while the expected price conditional on purchase receives more weight as n rises. Second, it has been proven above that $\alpha_{nk,1}/(1 - \alpha_{nk,0})$ decreases with n such that $1 \leq n \leq N - k + 1$. Then, these two effects must clearly cause a decrease in P_k^n as n rises. This proves that the virtual expected price falls, or that buyers' welfare rises, with greater product availability.

For (b), to prove that the expected price conditional on observing at least one price does not change with grater product availability for $j \geq N-k+2$, we need to demonstrate that

$$\frac{\alpha_{nk,1}}{1 - \alpha_{nk,0}} - \frac{\alpha_{n+1k,1}}{1 - \alpha_{n+1k,0}} = 0$$

for $n \geq N-k+2$. The equality holds as it is easy to check that $\alpha_{nk,1} = 0$ for $n \geq N-k+2$. Using similar steps as in the prove of (a), we can show that P_k^n does not change with $n \geq N - k + 2$. This means that buyers' welfare does not change with greater product availability for $j \geq N - k + 2$.

Proof of (ii) follows directly from discussion after the proposition in the main body of the paper.

The proof of the proposition is complete.

A.8 Proof of Proposition 8

We will use the following facts to prove the proposition. First, $E[p] = v + \int_{\underline{p}}^{v}(1-x_2(p))dp$. Second, $E[\min\{p_1,p_2\}] = v - 2\int_{\underline{p}}^{v}(1-x_2(p))dp + \int_{\underline{p}}^{v}(1-x_2(p))^2dp$, $k \neq l$ while

$$\begin{aligned} E[p] - E[\min\{p_1,p_2\}] &= \int_{\underline{p}}^{v}(1-x_2(p))dp - \int_{\underline{p}}^{v}(1-x_2(p))^2dp \\ &= v\mu(q)\left((1+2\mu(q))\ln\left(1+\frac{1}{\mu(q)}\right)-2\right). \end{aligned}$$

Then, we can rewrite (6) as

$$\frac{c}{v} = \theta_2\frac{2}{N}\mu(q)\left((1+2\mu(q))\ln\left(1+\frac{1}{\mu(q)}\right)-2\right). \tag{A.11}$$

The RHS of the equation is positive only if $(1+2\mu(q))\ln\left(1+\frac{1}{\mu(q)}\right) > 2$, or $\ln\left(1+\frac{1}{\mu(q)}\right) > \frac{2}{1+2\mu(q)}$. Note that when $\mu(q)\downarrow 0$ the LHS of the inequality goes to infinity while its RHS converges to 2, and when $\mu(q)\to\infty$ both the LHS and the RHS converge to 0. Then, it suffices to show that the derivative of LHS is more negative than that of the RHS for the inequality to hold. Indeed, the derivative of the LHS is $-\frac{1}{\mu(q)(1+\mu(q))} = -\frac{1+4\mu(q)+4\mu(q)^2}{\mu(q)(1+\mu(q))(1+2\mu(q))^2}$ while that of the RHS is $-\frac{4}{(1+2\mu(q))^2} = -\frac{4\mu(q)+4\mu(q)^2}{\mu(q)(1+\mu(q))(1+2\mu(q))^2}$, and the former is more negative than the latter. Summing up, this proves that the RHS of (A.11) is positive.

Now, we show that the RHS of (A.11) is inverse U-shaped in q, which is true only if it is inverse U-shaped which respect $\mu(q)$ as $\mu(q)$ is increasing in q with $\mu(0)=0$ and $\mu(1) = (1-\lambda)/(N-2+2\lambda)$. The derivative of the RHS with respect to $\mu(q)$ is

$$\begin{aligned} &\frac{2\theta_2}{N}\left((1+2\mu(q))\ln\left(1+\frac{1}{\mu(q)}\right)-2+\mu(q)\left[2\ln\left(1+\frac{1}{\mu(q)}\right)-\frac{1+2\mu(q)}{\mu(q)+\mu(q)^2}\right]\right) \\ = \;&\frac{2\theta_2}{N}\left(\frac{(1+5\mu(q)+4\mu(q)^2)\ln\left(1+\frac{1}{\mu(q)}\right)-3-4\mu(q)}{1+\mu(q)}\right), \end{aligned}$$

which is equal to zero only if its numerator is zero, or $M(\mu(q)) \equiv \ln\left(1+\frac{1}{\mu(q)}\right) - \frac{3+4\mu(q)}{(1+\mu(q))(1+4\mu(q))} = 0$. Thus, if $M(\mu(q)) = 0$ for only a single $\mu(q)$, then the RHS of (A.11) has one stationary point in $q\in(0,1)$. The following facts, along with the fact that $M(\mu(q))$ is continuous in $\mu(q)>0$, prove that $M(\mu(q))=0$ has a single solution in $\mu(q)\in(0,1)$:

$$M(0) = +\infty, \text{ and } M(\infty) = 0,$$

$$\frac{\partial M(\mu(q))}{\partial\mu(q)} = \frac{2\mu(q)-1}{\mu(q)(1+\mu(q))^2(1+4\mu(q))^2}\begin{cases} <0 \text{ for } \mu(q)<\frac{1}{2}, \\ =0 \text{ for } \mu(q)=\frac{1}{2}, \\ >0 \text{ for } \mu(q)>\frac{1}{2}. \end{cases}$$

Thus, the RHS of (A.11) has a unique stationary point in $q\in(0,1)$. To see that the RHS is maximized at that stationary point, note that the derivative of the RHS is positive for some values of q close to 0 and negative for some values of q close to 1. This completes the proof that the RHS is inverse U-shaped and has a unique maximum in $q \in (0,1)$.

Next, we demonstrate that the RHS of (A.11) is less than 1. For that, we rewrite the RHS as

$$\begin{aligned}\frac{c}{v} &= \frac{2}{N}\theta_2\left(\mu(q)\ln\left(1+\frac{1}{\mu(q)}\right)+2\mu(q)^2\ln\left(1+\frac{1}{\mu(q)}\right)-2\mu(q)\right)\\ &= \frac{2}{N}\theta_2\left\{\mu(q)\ln\left(1+\frac{1}{\mu(q)}\right)-2\mu(q)\left[1-\mu(q)\ln\left(1+\frac{1}{\mu(q)}\right)\right]\right\}.\end{aligned}$$

Next, we note that the first term in the large brackets, which is positive, is increasing in $\mu(q)$ and converges to 1 as $\mu(q)\to\infty$. Since the terms in the square brackets is positive and that the RHS is positive, the RHS must be less than 1.

Finally, we show that the RHS of (A.11) converges to zero as $q \to 0$, or $\mu(q)\to 0$. Note that since

$$\lim_{\mu(q)\to 0}\mu(q)\ln\left(1+\frac{1}{\mu(q)}\right) = \lim_{z\to\infty}\frac{\ln(1+z)}{z} \overset{\text{l'Hopital}}{=} \lim_{z\to\infty}\frac{1}{1+z}=0,$$

we have

$$\lim_{\mu(q)\to 0}\left(\mu(q)\ln\left(1+\frac{1}{\mu(q)}\right)-2\lim_{\mu(q)\to 0}\mu(q)\left[1-\lim_{\mu(q)\to 0}\mu(q)\ln\left(1+\frac{1}{\mu(q)}\right)\right]\right)=0.$$

Therefore, the RHS of (A.11) converges to zero as $q \to 0$.

The facts that, for $0<q<1$ the RHS of (A.11) is positive for, inverse U-shaped in q, and converges to zero as $q\to 0$, there must be a stable SBNE for sufficiently small search cost. The fact that the RHS (A.11) is less than one proves that $\bar{c}<v$.

A.9 Proof of Proposition 9

Parts (i) and (ii) are straightforward which is why they are omitted.

We prove part (iii) by showing that costly buyers are better off when θ_2 rises at the expense of θ_j, where $j \geq 3$. This is the case if

$$\frac{d}{d\theta_2}\left[\theta_2\left(q\frac{2}{N}E[p]+q\frac{(N-2)}{N}E[\min\{p_1,p_2\}])+(1-q)E[\min\{p_1,p_2\}]\right)+(N-1-q)c\right]<0.$$

Using (6) to substitute the value of c, we obtain

$$\frac{d}{d\theta_2}\left[\theta_2\left(E[\min\{p_1,p_2\}])+\left(2-\frac{2}{N}\right)(E[p]-E[\min\{p_1,p_2\}])\right)\right]<0. \tag{A.12}$$

To differentiate the LHS of the inequality, we note that, in equilibrium it must be that

$$\frac{d}{d\theta_2}\left[\frac{2}{N}\theta_2\left(E[p]-E[\min\{p_1,p_2\}]\right)\right]=0, \tag{A.13}$$

or

$$E[p] - E[\min\{p_1, p_2\}] + \theta_2 \frac{\partial(E[p] - E[\min\{p_1, p_2\}])}{\partial \mu(q)} \times \frac{d\mu(q)}{d\theta_2} = 0,$$

so that

$$\frac{d\mu(q)}{d\theta_2} = -\frac{E[p] - E[\min\{p_1, p_2\}]}{\theta_2 \frac{\partial(E[p]-E[\min\{p_1,p_2\}])}{\partial \mu(q)}}. \tag{A.14}$$

Now, using (A.13), we rewrite (A.12) as

$$\frac{d\theta_2 E[\min\{p_1, p_2\}]}{d\theta_2} < 0,$$

so

$$\theta_2 \left(E[\min\{p_1, p_2\}] + \frac{\partial E[\min\{p_1, p_2\}]}{\partial \mu(q)} \times \frac{d\mu(q)}{d\theta_2} \right) < 0.$$

We substitute the value of $d\mu(q)/d\theta_2$ from (A.13) and simplify to obtain

$$\frac{\partial E[p]}{\partial \mu(q)} E[\min\{p_1, p_2\}] - \frac{\partial E[\min\{p_1, p_2\}]}{\partial \mu(q)} E[p] < 0.$$

Employing the expression for $E[p]$ and $E[\min\{p_1, p_2\}]$, we obtain

$$v \frac{2\mu(q) \left(\mu(q)(1 + \mu(q)) \ln^2 \left(1 + \frac{1}{\mu(q)}\right) - 1 \right)}{1 + \mu(q)} < 0, \tag{A.15}$$

which is true only if the terms in the large brackets in the numerator are negative. Namely,

$$\ln^2 \left(1 + \frac{1}{\mu(q)}\right) < \frac{1}{\mu(q)(1 + \mu(q))},$$

or

$$-\frac{1}{\sqrt{\mu(q)(1 + \mu(q))}} < \ln \left(1 + \frac{1}{\mu(q)}\right) < \frac{1}{\sqrt{\mu(q)(1 + \mu(q))}}.$$

The left-hand inequality clearly holds for any $\mu(q) > 0$. Regarding the right-hand inequality, as both its sides go to infinity as $\mu(q) \to 0$ and converge to 0 as $\mu(q) \to \infty$, the inequality holds if the derivative of the LHS is less negative than that of the RHS. The derivative of the LHS is equal to $-\frac{1}{\mu(q)+\mu(q)^2}$ and that of the RHS is $-\frac{1}{\mu(q)+\mu(q)^2} \times \frac{1+2\mu(q)}{2\sqrt{\mu(q)+\mu(q)^2}}$. The former is less negative than the latter if $1 + 2\mu(q) < 2\sqrt{\mu(q) + \mu(q)^2}$, or

$$1 + 4\mu(q) + 4\mu(q)^2 > 4\mu(q) + 4\mu(q)^2$$

which is certainly true. This establishes that (A.15) holds. This means that costly buyers are better off with an increase θ_2 at the expense of θ_j for $j \neq 3$. The proof is complete.

A.10 Proof of Proposition 10

To prove the proposition, we first start with some immediate results. Specifically, for $\rho < v$, we have the properties of price distributions stated in the following lemma.

Lemma 4. *It must be that (i) $x_n(\min\{\rho, v\}) = 0$, (ii) $x_n(p)$ does not have atoms or (iii) flat regions in the support for $n = 2, 3$.*

Proof of Lemma 4. We prove all parts of the lemma by contradiction.

(i) If a seller sets a price greater than v, it clearly does not make any sales. For $\rho \leq v$, if $x_n(\rho) = 0$, a seller that sets a price greater than ρ does not sell to anyone. Obviously, costless consumers who observe all prices do not buy from the seller under question as the rival sellers' price is definitely lower than ρ. For costly consumers, it may happen that a consumer obtains the price of the seller under question first. Then, she definitely searches as the seller's price is greater than ρ. Therefore, the consumer observes prices of all other sellers and makes purchase from one of those rival sellers. It may also happen that a costly consumer learns a price of one of the rival sellers first. Since all the rival sellers price below ρ, the consumer makes a purchase outright. Finally, if a consumer happens to visit an inactive firm (which happens only when $n = 2$ is realized), she searches for prices at a price aggregator if $\rho < v$ or is indifferent between searching and dropping out of the market if $\rho = v$. Clearly, in either case she does not make purchase form the seller under question. Hence, pricing greater than ρ yields zero profit, whereas pricing below ρ yields a strictly positive profit in expectation, a contradiction.

(ii) Suppose there is an atom at price $\widetilde{p}$. Then, an individual seller prefers to slightly undercut the price as it yields a discontinuous increase in the demand due to the strictly positive share consumers who compare prices, a contradiction.

(iii) If $x_n(p)$ has flat a region in the support, the expected demand for prices in that region is constant. Then, an individual seller cannot be different of charging any price in that flat region—the highest price in the region yields the highest payoff, a contradiction. □

Now, it is left to determine ρ, show its uniqueness, and that $\rho < v$ only for $c < \bar{c}$ where $\bar{c} \in (0, v)$. For that, we first determine a costly consumer's posterior belief regarding realization of $n = 2$, denoted by $\omega(p)$, after observing price p at the first seller. The consumer updates her belief according to Bayes' rule:

$$\omega(p) = \frac{\frac{2}{3}\theta_2 x_2'(p)}{\frac{2}{3}\theta_2 x_2'(p) + \theta_3 x_3'(p)}. \tag{A.16}$$

Correspondingly, at price ρ, the consumers belief regarding the realization of $n = 2$ is

$$\omega(\rho) = \frac{\frac{2}{3}\theta_2 x_2'(\rho)}{\frac{2}{3}\theta_2 x_2'(\rho) + \theta_3 x_3'(\rho)} = 0,$$

where the last equality follows from the fact that $|x_2'(\rho)| < \infty$ and $|x_3'(\rho)| = \infty$. Then, the reservation price solves $\rho = \mathbb{E}_{x_3(p)}[\min\{p_i, p_j\}] + c$. We use the fact that the CDF of minimum of two prices is $1 - x_3(p)^2$ to write $\mathbb{E}_{x_3(p)}[\min\{p_i, p_j\}] = \int_{\frac{(1-\lambda)\rho}{1+2\lambda}}^{\rho} p d(1 - x_3(p)^2)$. We apply integration by parts and some algebraic manipulations to obtain (9).

Clearly, as the LHS of (9) is strictly increasing in ρ while the RHS is independent of ρ, there must be a unique ρ that solves the equation for $c > 0$ if the solution exists. As ρ is strictly increasing in c, there is a unique value of c such that $\rho = v$ for any $\theta_2 \in (0, 1)$ and $\theta_2 + \theta_3 = 1$. This cutoff value of c, denoted by $\bar{c}$, is less that v because the LHS of (9) is less than v.

To complete the proof, we need to show that, when there is discontinuity in the posterior belief of a costly consumer, her search behavior is not affected by it. First, we show the existence of the discontinuity. The discontinuity in the posterior belief exists only if the lower bound of the support of $x_2(p)$ is lower than that of $x_3(p)$, which is true if $\frac{1-\lambda}{2+\lambda} < \frac{1-\lambda}{1+2\lambda}$, or $1+2\lambda < 2+\lambda$, which reduces to $\lambda < 1$, which certainly holds. This demonstrates that the lower bound of the support of $x_2(p)$ is indeed lower than that of $x_3(p)$. Then, a costly consumer that observes a price equal to $\left(\frac{1-\lambda}{1+2\lambda}\right)\rho - \epsilon$, where $\epsilon > 0$ is a arbitrarily small, knows for sure that there are two sellers in the market. This consumer does not search if

$$\int_{\left(\frac{1-\lambda}{2+\lambda}\right)\rho}^{\left(\frac{1-\lambda}{1+2\lambda}\right)\rho} (1 - x_2(p))dp < c, \tag{A.17}$$

where we used similar notion and techniques as those we applied to obtain (9). We replace c by the LHS of (9) in the above inequality to obtain

$$\int_{\left(\frac{1-\lambda}{2+\lambda}\right)\rho}^{\left(\frac{1-\lambda}{1+2\lambda}\right)\rho} (1 - x_2(p))dp \leq \int_{\left(\frac{1-\lambda}{1+2\lambda}\right)\rho}^{\rho} (1 - [x_3(p)]^2)dp.$$

We apply price distributions in (7) and (8) to evaluate the integrals and rewrite the inequality as

$$\frac{1-\lambda}{1+2\lambda}\left(\frac{1-\lambda}{1+2\lambda} - \ln\left(\frac{2+\lambda}{1+2\lambda}\right)\right) \leq 1 - \frac{1-\lambda}{2\lambda}\ln\left(1 + \frac{3\lambda}{1-\lambda}\right), \tag{A.18}$$

where ρ on both sides of the inequality cancel out. We note that in the limit as $\lambda \to 0$, the LHS converges to $1-\ln(2)$ and as $\lambda \to 1$ it approaches 0. When $\lambda \to 0$ the RHS converges to 0 and as $\lambda \to 1$ it converges to 1 (where we used a similar method as in the proof of Proposition 8 to evaluate the limits). Then, there is a unique value of λ such that the LHS and the RHS of (A.18) are equal if the LHS is decreasing and the RHS is increasing in λ. The derivative of the LHS with respect to λ is $-\frac{9(1-\lambda)-3(2+\lambda)(1+2\lambda)\ln\left(\frac{2+\lambda}{1+2\lambda}\right)}{(2+\lambda)(1+2\lambda)^3}$. As its the denominator is positive, the derivative is negative if $\ln\left(\frac{2+\lambda}{1+2\lambda}\right) < \frac{3(1-\lambda)}{(2+\lambda)(1+2\lambda)}$. As the LHS and the RHS of the inequality respectively converge to $\ln(2)$ and $3/2$ when $\lambda \to 0$, while both sides converge to 0 when $\lambda \to 1$, the inequality holds if the derivative of the LHS is less negative than that of the RHS. The derivative of the LHS is $-\frac{6+15\lambda+6\lambda^2}{(2+\lambda)^2(1+2\lambda)^2}$ and that of the RHS is $-\frac{21+12\lambda-6\lambda^2}{(2+\lambda)^2(1+2\lambda)^2}$, and the former is indeed less negative than the latter as the numerator of the former is less than the latter for $\lambda \in (0,1)$. Then, the inequality holds, meaning that the derivative of the LHS of (A.18) with respect to λ is negative.

Next, the derivative of the RHS of (A.18) with respect to λ is positive if its derivative with respect to ϕ, where $\phi \equiv (1-\lambda)/(3\lambda)$, is negative. The latter derivative is $\frac{1}{1+\phi} - \ln\left(1 + \frac{1}{\phi}\right)$. Observe that as $\phi \to 0$, the expression goes $-\infty$, while as $\phi \to \infty$ it converges to 0. Then, the expression is negative if its derivative w.r.t. ϕ is positive. This derivative is equal to $\frac{1}{\phi(1+\phi)^2}$, which is clearly positive for $\phi > 0$. Then, the derivative of the RHS of (A.18) is negative with respect to ϕ, which means that the RHS is increasing in λ.

Summing up, the LHS of (A.18) is decreasing in $\lambda \in (0,1)$, converges to $1 - \ln(2)$ when $\lambda \to 0$ and approaches 0 when $\lambda \to 1$; whereas the RHS of the (A.18) is increasing in λ, converges to 0 when $\lambda \to 0$ and to ∞ when $\lambda \to 1$. Then, there must be a unique

$\overline{\lambda} \in (0,1)$ such that the inequality in (A.18) holds for $\lambda \geq \overline{\lambda}$. Then, the SRPE exists only if (A.18) holds, which is true if $\lambda \geq \overline{\lambda}$.

This completes the proof of the proposition.

A.11 Proof of Proposition 11

For the proof, it suffices to show that the price distribution in the triopoly market first order stochastically dominates that in the duopoly market:

$$x_2(p) < x_3(p) \text{ for all } p \in \left[\frac{(1-\lambda)\overline{p}}{1+2\lambda}, \overline{p}\right),$$

where $\overline{p} = \min\{\rho, v\}$. We rewrite the inequality as

$$\begin{aligned}
\left[\frac{1-\lambda}{1+2\lambda}\left(\frac{\overline{p}}{p}-1\right)\right]^2 \quad &< \quad \frac{1-\lambda}{3\lambda}\left(\frac{\overline{p}}{p}-1\right), \\
\frac{3\lambda(1-\lambda)}{(1+2\lambda)^2}\left(\frac{\overline{p}}{p}-1\right) \quad &< \quad 1.
\end{aligned}$$

As the LHS of the inequality is decreasing in p, the inequality certainly holds if the following is true:

$$\begin{aligned}
\frac{3\lambda(1-\lambda)}{(1+2\lambda)^2}\left(\frac{\overline{p}}{\frac{(1-\lambda)\overline{p}}{1+2\lambda}}-1\right) \quad &< \quad 1 \\
\left(\frac{3\lambda}{1+2\lambda}\right)^2 \quad &< \quad 1,
\end{aligned}$$

which in its turn holds if

$$\frac{3\lambda}{1+2\lambda} < 1,$$

which is true for any $\lambda \in (0,1)$. This means that $x_2(p) < x_3(p)$ for all p in the support of $x_3(p)$. The proof is complete.

References

Anderson, Simon P., Andre De Palma, and Jean Francois Thisse, *Discrete Choice Theory of Product Differentiation*, The MIT Press, 1992.

Armstrong, Mark, John Vickers, and Jidong Zhou, "Consumer Protection and Incentives to Become Informed," *Journal of the European Economic Association*, 2009, *7*, 399–410.

Benabou, Roland and Robert Gertner, "Search with Learning from Prices: Does Increased Inflationary Uncertainty Lead to HigherMarkups," *Review of Economic Studies*, 1993, *60* (1), 69–93.

Brown, Jeffrey and Austan Goolsbee, "Does the Internet Make Markets More Competitive? Evidence from the Life Insurance Industry," *Journal of Policitcal Economy*, 2002, *110* (3), 481–507.

Burdett, Kenneth and Kenneth Judd, "Equilibrium Price Dispersion," *Econometrica*, 1983, *51* (4), 955–969.

Chen, Yongmin and Michael Riordan, "Price-Increasing Competition," *RAND Journal of Economics*, 2008, *39* (4), 1042–1058.

_ **and Tianle Zhang**, "Entry and Welfare in Search Markets," *The Economic Journal*, 2018, *128*, 55–80.

Dana, James, "Learning in an Equilibrium Search Model," *International Economic Review*, 1994, *35* (3), 745–771.

Diamond, Peter, "A Model of Price Adjustment," *Journal of Economics Theory*, 1971, *3*, 156–168.

Ellison, Glenn and Sarah Fisher Ellison, "Search, Obfuscation, and Price Elasticities on the Internet," *Econometrica*, 2009, *77* (2), 427–452.

Fershtman, Chiam and Arthur Fishman, "The "Perferse" Effects of Wage and Price Controls in Search Markets," *European Economic Review*, 1994, *38*, 1099–1112.

Fishman, Arthur and Nadav Levy, "Search Costs and Investment in Quality," *The Journal of Industrial Economics*, 2015, *53* (4), 625–641.

Gabaix, Xavier, David Laibson, Deyuan Li, Hongyi Li, Sidney Resnick, and Casper G. de Vries, "The Impact of Competition on Prices with Numerous Firms," *Journal of EconomicTheory*, 2016, *165*, 1–24.

Gilad, Benjamin, "Companies Collect Competitive Intelligence, but Don't Use It," 2015. Last accessed on April 13, 2020 at https://hbr.org/2015/07/companies-collect-competitive-intelligence-but-dont-use-it.

Gomis-Porqueras, Pedro, Benoit Julien, and Chengsi Wang, "Strategic Advertising and Directed Search," *International Economic Review*, 2017, *58* (3), 783–806.

Hey, John D. and Chris J. McKenna, "Consumer Search with Uncertain Product Quality," *Journal of Political Economy*, 1981, *9* (1), 54–66.

Hong, Han and Matthew Shum, "Using Price Distributions to Estimate Search Costs," *RAND Journal of Economics*, 2006, *37* (2), 257–275.

Honka, Elisabeth and Pradeep Chintagunta, "Simultaneous or Sequential? Search Strategies in the U.S. Auto Insurance Industry," *Marketing Science*, 2017, *36* (1), 21–42.

Janssen, Maarten and Eric Rasmusen, "Bertrand Competition under Uncertainty," *The Journal of Industrial Economics*, 2002, *50*, 11–21.

_ **and Jose Luis Moraga-Gonzalez**, "Strategic Pricing, Consumer Search and the Number of Firms," *Review of Economic Studies*, 2004, *71*, 1089–1118.

_ **and Marielle Non**, "Going Where the Ad Leads You: On High Advertised Prices and Searching Where to Buy," *Marketing Science*, 2009, *28* (1), 87–98.

_ **, Paul Pichler, and Simon Weidenholzer**, "Oligopolistic Markets with Sequential Search and Production Cost Uncertainty," *RAND Journal of Economics*, 2011, *42* (3), 444–470.

Johnen, Johannes and David Ronayne, "On the Simple Economics of Advertising, Marketing, and Product Design," *Working Paper*, 2020.

Lester, Benjamin, "Information and Prices with Capacity Constraints," *American Economic Review*, 2011, *101* (4), 1591–1600.

los Santos, Babur De, Ali Hortaçsu, and Matthijs R. Wildenbeest, "Testing Models of Consumer Search Using Data on Web Browsing and Purchasing Behavior," *American Economic Review*, 2012, *102* (6), 2955–2980.

MacMinn, Richard D., "Search and Market Equilibrium," *Journal of Political Economy*, 1980, *88* (2), 308–327.

Moraga-Gonzalez, Jose Luis, Zsolt Sandor, and Matthijs Wildenbeest, "Non-sequential Search Equilibrium with Search Cost Heterogeneity," *International Journal of Industrial Organization*, 2017, *50*, 392–414.

_ **,** _ **, and** _ , "Simultaneous Search for Differentiated Products: the Impact of Search Costs and Firm Prominence," *Working Paper*, 2018.

Parakhonyak, Alexei and Anton Sobolev, "Non-reservation Price Equilibrium and Search without Priors," *The Economic Journal*, 2015, *125* (584), 887–909.

Perloff, Jeffrey M. and Steven C. Salop, "Equilibrium with Product Differentiation," *Review of Economic Studies*, 1985, *52* (1), 107–120.

Pesendorfer, Wolfgang and Asher Wolinsky, "Second Opinions and Price Competition: Inefficiency in the Market for Expert Advice," *Review of Economic Studies*, 2003, *70* (2), 417–437.

Rhodes, Andrew, "Can Prominence Matter even in an Almost Frictionless Market?," *The Economic Journal*, 2011, *121* (556), F297–F308.

Stahl, Dale O.II, "Oligopolistic Pricing with Sequential Consumer Search," *American Economic Review*, September 1989, *79* (4), 700–712.

Stigler, George, "The Economics of Information," *Journal of Political Economy*, 1961, *69* (3), 213–225.

Tappata, Mariano, "Rockets and Feathers: Understanding Asymmetric Pricing," *The RAND Journal of Economics*, 2009, *40* (4), 673–687.

Varian, Hal R., "A Model of Sales," *American Economic Review*, September 1980, *70* (4), 651–659.

Wolinsky, Asher, "Procurement via Sequential Search," *Journal of Political Economy*, 2005, *113* (4), 785–810.

Wright, Randall, Philipp Kircher, Benoit Julien, and Veronica Guerrieri, "Directed Search: a Guided Tour," *NBER Working Paper 23884*, 2017.

UniCredit Foundation

Piazza Gae Aulenti, 3

UniCredit Tower A

20154 Milan

Italy

Giannantonio De Roni – *Secretary General*

e-mail: giannantonio.deroni@unicredit.eu

Annalisa Aleati - *Scientific Director*

e-mail: annalisa.aleati@unicredit.eu

Info at:

www.unicreditfoundation.org

UniCredit Foundation